APOLLONII CONICA:

Methodo Nova Illuſtrata, & Succinctè Demonstrata.

Per ISAACUM BARROW,
Ex-profeſſorem Lucaſianum *Cantab.* & Soc. Regiæ Soc.

LONDINI,
Excudebat *Guil. Godbid,* vœneunt apud *Robertum Scott,*
in vico Little-Britain, 1675.

APOLLONII
CONICA.

De Apollonio Pergæo, ex Vossio de Scriptoribus Mathematicis:

Proximæ post *Euclidem* antiquitatis, ex iis qui extant est Apollonius Pergæus, Claruit enim sub Ptolomæo Evergete.

N*Atus est Pergæ, civitate Pamphiliæ. De ætate autem ejus, quod dixi auctor est* Heraclius *in* Archimedis *vita, & exinde* Eutocius Ascalonita *initio commentariorum in Conica* Apollonii. *Ubi ex* Gemini *Mathematicarum præceptionum libro sexto refert, ut ob conicam hanc scientiam à suæ ætatis hominibus nuncupatus sit* magnus Geometra, Euclidis *discipulos, quod & ipsum ætatem indicat,* Alexandriæ *audivit, à quibus cum multa accepisset; non difficile adeò fuit quatuor conicorum* Euclidis *libros commentario illustrare, ac totidem alios adjungere: ut in universum conicorum essent libri viij: sicut auctor est Pappus* Alexandrinus *libro vij Mathematicæ Collectionis.* Pag. 54.

Atque eodem libro alia quoque ejusdem Apollonii *memorat, videlicet libros duos* περὶ λόγου ἀποτομῆς, de proportionis sectione: *totidem* περὶ χωρίου ἀποτομῆς, de spatii sectione: *etiam duos* διωρισμένης τομῆς, determinatæ sectionis: *ac totidem* ἐπαφῶν, tactionum: *duos quoque* νεύσεων, inclinationum: *ac similiter duos* τόπων ἐπιπέδων, planorum locorum. *Quorum* Pappus *alios vocat* ἐφεκτικοὺς, in se solis consistentes: *alios* διεξοδικοὺς, extra sese tendentes. Pag. 55.

Cæterùm fuere olim, qui Conica non esse Apollonii Pergæi, *sed* Archimedis *putarent. Existimavit hoc* Heraclius *ille, qui* Archimedis *vitam retulit. Ejus verba inde adducit* Eutocius, *quibus ait* Archimedem *primum omnium consignasse* Elementa Conica: Apollonium *autem, cùm ea sciret necdum edita esse ab auctore, involâsse in illa, proque suis edidisse. Ac videtur id posse ipsius* Archimedis *verbis comprobari. Nam ipse operis de Conicis Elementis meminit partim libro* περὶ κωνοειδέων, καὶ σφαιροειδέων, *ante propositionem quartam. Utrobique enim ait, id, de quo loquitur demonstratum esse in libris Conicis. Nisi quis censeat, respici* Euclidis *quatuor Conicorum libros: quorum* Pappus *mentionem facit in septimo Mathematicarum Collectionum libro. Verum, si alienum opus signaret* Archimedes *non sic loqueretur. Solet enim tum dicere, priores id demonstrasse. Et sanè* Archimedi *non fuisse ignotum conos secari posse planis, quæ ad latus Coni habeant inclinationem differentem, satis comprobat contra* Eutocium, *& alios,* Guido Ubaldus *initio Commentarii in secundum* ἰσοῤῥοπικῶν Archimedis. *Quæ cùm considero, nolim de auctore multùm contendere. Et fortassa rudiores* Archimedis *chartas nactus fuit* Apollonius, *atque eas perfecit. Utcunque est, ante hæc de Conicis edita ab* Apollonio, *imperfecta erat eorum notitia: ut tradit* Eutocius Ascalonita Pag. 434.

initio

initio Commentarii ſui in Conica Apollonii. Atque, Ariſtæi judicio, Pappi eadem mens fuit.

Quatuor priores libros primus tranſtulit Joan. Baptiſta Memmius; ſed infeliciter, eò quòd Argumentum operis non intelligeret: unde, non vidit ſat manifeſtas Græci codicis mendas, ac ſæpe pueriliter hallucinatur: ſicut monitum Franciſco Maurolyco, præfatione in Coſmographiam ſuam. Opera igitur fecit Federicus Commandinus; qui demùm Latinè vertit, Bononiæque edidit anno 1566. Nec tamen omnia in eo vertendo potuit Commandinus. Uſque adeo corruptus erat Græcus codex, quare in Latinis vitioſum ſequi codicem eſt coactus; ut ipſe agnoſcit.

Alia quoque Apollonii hujus citant Proclus in Euclidem, ubi laudatur Ἀπολλώνιος ὁ Περγαῖος ἐν τῷ Ὠκυτοκίῳ. Fortaſſe reſcribendum τῷ Ὠκυβόλῳ. Quæ vox fuerat à βολὴ ſignificante pugnam: ut ſæpe apud Homerum: Ὠκυβόλοι, ταχυμάχαι. Et quædam recenſet Eutocius Aſcalonita libro de dimenſione Circuli, Eutocius his etiam in Conica Apollonii commentatus fuit. Eum quoque commentarium Latinè vertit Commandinus. Utcunque verò nunc *quatuor duntaxat Conicorum libros habeamus, Arabicè tamen tres præterea ex Oriente eſt nactus Clariſſimus Jacobus Golius, antehac, in Academia Leidenſi, collega conjunctiſſimus. Cui multum Arabica debent literæ ac Matheſis univerſa: plura verò propediem debebunt; preſertim ubi tres illi Conicorum libri viderint lucem. Quærat aliquis quid factum ſit de libro octavo, Cognoſcimus de hoc ex Codice Goliano: ubi ad Calcem erat adſcriptum eo non fuiſſe Arabicè tranſlatum; quia etiam liber ille deſideraretur in Codicibus Græcis, unde Arabes cætera tranſtuliſſent. Sed doctiſſimus Merſennius, præfatione in Apollonii Conica; quæ ſunt in ejus Συνοψιν Mathematica; Arabicè illum extare ait: imò omnes Apollonii libros eâ linguâ legi; ſane plures etiam, quam enumerarit Pappus. Atque horum teſtem citat Aben Nedin; qui librum contexuit de Philoſophis Arabibus, omniumque eorum ſcripta memoravit, qui fuere à quadringenteſimo poſt Muhammedem anno. Interim (ut idem Merſennius addit) Claudii Mydorgii, patricii Pariſini, ea eſt ſuſpicio tres illos Conicorum libros, qui ab Arabibus Apollonii credumtur non eſſe genuinos, verùm ab aliquo ſuppoſitos, qui ſub Apollonii nomine facere voluerit fucum. Atque hoc inde colligit; quod libro quinto prima Propoſit. in vj. Apollonii non tantummodo in Cono recto, ſed in Scaleno etiam quolibet, & portionibus quibuſvis, demonſtret poſſibilia quæque. Meminit auctoris quoque Vitruvius, lib. 1. cap. 1. Cardanus in 16 de Subtilitate ei ſeptimum inter ſubtilia orbis ingenia tribuit locum.

*Anno 1650. antequam tres poſteriores à Borellio ederentur.

APOL-

APOLLONII CONICORUM LIB. I.

DEFINITIONES.

I.

SI ab aliquo puncto (A) ad circumferentiam Circuli (BHC,) qui non sit in eodem plano, in quo punctum, conjuncta recta linea (AB) in utramque partem producatur, & manente puncto (A) convertatur circa circuli circumferentiam, quousque ad eum locum redeat, à quo cœpit moveri; superficiem (DAEFG) à recta linea descriptam, constantémq; ex duabus superficiebus (DAG, EAF) ad verticem (A) inter se aptatis, quarum utraque in infinitum augetur, nimirum recta linea (EABD) quæ eam describit in infinitum aucta, voco *Conicam superficiem.* Fig. 1.

II.

Verticem ipsius, manens punctum (A).

III & VI.

Axem, rectam lineam (AG), quæ per punctum (A), & circuli centrum (G) ducitur.

IV.

Conum autem voco figuram (ABC), contentam circulo (BHC), & conicâ superficie (BAC), quæ inter verticem, & circuli circumferentiam interjicitur.

V & VII.

Baſim, circulum ipſum (B H C).

VIII.

Conorum rectos voco, qui axes habent ad rectos angulos ipſis baſibus.

IX.

Scalenos, qui non ad rectos angulos ipſis baſibus axes habent.

X.

Fig. 2. Omnis curvæ lineæ (A B C) in uno plano exiſtentis *Diametrum* voco, rectam lineam (B D,) quæ quidem ducta à linea curva, omnes lineas (A C), quæ in ipſa ducuntur cuidam lineæ (A C) æquidiſtantes, bifariam dividit.

XI.

Verticem lineæ rectæ terminum (B,) qui eſt in ipſa linea (A B C.)

XII.

Ordinatim ad diametrum *applicari* dicitur, unaquæque æquidiſtantium linearum (A C.)

XIII.

Fig. 3. Similiter, & duarum curvarum linearum (C A D, E B F) in uno plano exiſtentium, *diametrum* quidem *tranſverſam* voco, rectam lineam (A B), quæ omnes (C D, E F) in utraque ipſarum ductas, rectæ cuidam (C D, vel E F) æquidiſtantes bifariam dividit.

XIV.

Vertices, diametri terminos (A, B) qui ſunt in ipſis lineis (C A D, E B F.)

XV.

Rectam verò *diametrum* (Z Y) voco, quæ inter duas lineas (C A D, E B F)

E B F) posita, lineas omnes (C E, D F) ductas rectæ cuidam (A B) æquidistantes, & inter ipsas interjectas, bifariam secat (in [illegible])

XVI.

Ordinatim ad diametrum applicari dicitur unaquæque linearum æquidistantium (C D, vel E F ad A B; & C E, vel D F ad Z Y.)

XVII.

Conjugatas diametros voco curvæ lineæ (A Z B Y), & duarum curvarum (C A D, E B F) rectas lineas (A B, Z Y), quarum utraque diameter est, & rectas lineas (C D, C E) bifariam dividit. Fig. 4.

XVIII.

Axem verò curvæ lineæ, & duarum curvarum, rectam lineam, quæ cùm sit diameter curvæ lineæ, vel duarum curvarum, æquidistantes ad rectos secat angulos.

XIX.

Axes conjugatos curvæ lineæ, & duarum curvarum, rectas lineas, quæ cùm sint diametri conjugatæ, ipsis æquidistantes ad rectos angulos secant.

Nihilo differunt axes à diametris, nisi quòd indifferenter illæ, hi ad rectos tantùm secent.

Apollonius Eudemo.

S. D.

SI & corpore vales, & aliæ tuæ res ex animi tui ſententia ſe habent, bene eſt; nos quidem ſatis bellè habemus. Quo tempore tecum *Pergami* fui, animadverti te cupidum intelligendi Conica, quæ à nobis conſcripta ſunt. Itaque miſi ad Te primum librum emendatum, reliquos deinceps miſſurus, cùm animo ero tranquilliori; non enim arbitror te oblitum quod à me accepiſti, quid ſcilicet cauſæ fuerit cur ego hæc ſcribere aggreſſus ſim; rogatus à *Naucrate* Geometrâ, quo tempore *Alexandriam* veniens apud nos fuit; & cur nos, cùm de illis octo libris egiſſemus, majorem ſtatim in his diligentiam adhibuimus. Nam cùm ipſe *Naucrates* quamprimum eſſet navigaturus, nos ea non emendavimus, ſed quæcunque ſeſe nobis obtulerunt conſcripſimus, utpote qui ea poſtremò eſſemus percurſuri. Quamobrem nunc tempus nacti, ut quæque emendamus, ita edimus. Et quoniam accidit nonnullos alios ex iis, qui nobiſcum fuerant, habuiſſe primum & ſecundum librum antequam emendaretur, noli mirari, ſi in quædam incidas, quæ aliter ſe habeant. Ex octo autem libris, quatuor primi hujus diſciplinæ continent Elementa: quorum primus complectitur generationes trium Coni ſectionum, & earum quæ oppoſitæ dicuntur, itemq; principalia ipſarum accidentia, à nobis & uberiùs & univerſaliùs, quàm ab iis qui de ea re ſcripſerunt elaborata. Secundus Liber tractat ea quæ attinet ad Diametros, & ad Axes Sectionum, & ad illas lineas *quæ cum ſectione non conveniunt; tum de aliis diſſerit, quæ & generalem & neceſſariam utilitatem ad determinationes afferunt. * Tertius liber continet multa & admirabilia

*ἀσυμπτώτους.

Theo-

Theoremata, quæ utilia erunt & ad ſolidorum locorum compoſitiones, & ad determinationes; quorum complura & pulcherrima, & nova ſunt. Hæc nos perpendentes animadvertimus, non poſitam eſſe ab *Euclide* rationem componendi loci ad tres & quatuor lineas, verùm ipſius tantummodo particulam quandam, atque hanc non ſatis feliciter. Neque enim fieri poterat, ut ea compoſitio rectè perficeretur abſque iis, quæ à nobis inventa ſunt. Quartus liber tradit quot modis Conorum ſectiones inter ſeſe, & circuli circumferentiæ occurrere poſſint, & multa alia ad pleniorem doctrinam, quorum nihil ab iis, qui ante nos fuerunt, memoriæ proditum eſt; coni ſectio, & circuli circumferentia, & oppoſitæ ſectiones ad quot puncta oppoſitis ſectionibus occurrant. Reliqui autem 4 Libri ad abundantiorem ſcientiam pertinent. Etenim Quintus de Minimis & Maximis magna ex parte agit. Sextus de Æqualibus, & ſimilibus Coni ſectionibus. Septimus continet Theoremata, quæ determinandi vim habent. Octavus Problemata Conica determinata. At verò omnibus his editis, licet unicuique, qui in ea legendo inciderit, ex animi ſui ſententia judicare. *Vale.*

LIB. I.

Prop. I.

REctæ lineæ (A G) quæ à vertice (A) ſuperficiei Conicæ ad puncta (G) quæ in ſuperficie ſunt, ducuntur, in ipſa ſuperficie erunt. Fig. 5.

Cùm enim puncta A G ſint in ſuperficie conica, (*a*) recta ipſam deſcribens per puncta A, G tranſibit. Itaque liquet tunc rectam A G in ſuperficie conica exiſtere. (*a*) 1. *def.* 1. *hujus.*

Coroll. 1. Hinc conſtat, ſi à vertice ad aliquod punctum eorum quæ ſunt intra ſuperficiem, recta linea ducatur, intra, & ſi ad aliquod eorum quæ ſunt extra, extra ſuperficiem cadere. *Cor.*

Coroll. 2. Rectæ à vertice ad puncta, quæ in superficie, ductæ, basis circumferentiæ occurrent, si opus est, protractæ.

Prop. II.

Si in alterutra superficierum, quæ sunt ad verticem, sumantur duo puncta (D,E), & quæ puncta conjungit recta linea (D E) ad verticem non pertineat, intra superficiem cadit, quæ verò est in directam (E F) cadet extrà.

Fig. 6.

Conjungantur rectæ A D, A E [a] occurrentes basis circumferentiæ punctis B,G, quæ connectat recta B G; hæc [b] intra circulum cadet, ac proinde intra superficiem conicam; ergo [c] planum trianguli A B G est intra superficiem conicam; ergo recta D E in ipso sita [c] est intra eandem. Porro recta A F [d] cadit in B G protractam extra circulum; & [e] proinde extra superficiem conicam; ergo punctum F est etiam extra ipsam.

a 2 *Cor.* 1. 2 *hujus.*
b 2. 3.
c 2. 11.
d 10. 3.
e 1. *Cor.* 1 *hujus.*

Prop. III.

Si conus (A B C) plano per verticem (A) secetur, sectio (A B C) triangulum erit.

Fig. 7.

Nam A B & A C [a] rectæ sunt: item B C [b] est recta. [c] ergo A B C est triangulum.

a 1. *hujus.*
b 3. 11.
c 20. *def.* 1.

Prop. IV.

Si alterutra superficierum, quæ sunt ad verticem, secetur plano (D H E) æquidistante circulo (B K C), per quem fertur recta linea superficiem describens, planum (D H E) quod superficie concluditur, circulus erit, habens centrum (G) in axe (A F); figura verò (A D E) contenta circulo (D H E) & ea parte superficiei conicæ, quæ inter secans planum (D H E), & verticem (A) interjicitur, conus erit.

Fig. 8.

Planum per axem A F faciat [a] trigonum A B C; communisque sectio ejus cum plano D H E [b] sit recta D E. In sectione D H E sumatur punctum utcunque H, ducaturque recta A H K [c] circumferentiæ basis occurrens in K, & connectantur G H, F K. Jam, ob D E, B C, ac G H, F K [d] parallelas, erit F B. G D [e] :: (A K. A G [e] ::) F K. G H. unde cum F B F K [f] æquentur, [g] erunt etiam G D, G H æquales, idémque de reliquis à G ad sectionem ductis rectis ostendi poterit. Unde

a 3. *hujus.*
b 3. 11.
c 2. *Cor.* 1. *hujus.*
d *hyp* & 16. 11.
e 4. 6.
f *hyp* & 15. *def.* 2.
g 14. 5.

de sectio DHE[h] circulus erit, cujus centrum G, & [k] proinde figura ADE conus.

h 15. def. 1.
k 4 def. hujus.

Coroll. 1. Recta DE est diameter circuli DHE. [l]

2. Conus ADE[m] similis est cono ABC (ob AG. GD :: AF. FB.)

l
m 24 def. 11.

Prop. V.

Si conus Scalenus secetur plano (ABC) per axem ad rectos angulos ipsi basi (BC), secetúrque altero plano (GHK) ad triangulum per axem recto, quod ex parte verticis (A) abscindat triangulum (AGK) simile (ei ABC) quod per axem, * subcontrariè vero positam; sectio (GHK) circulus erit. Vocetur autem Sectio subcontraria. Fig. 9.

In sectione GHK accipiatur punctum H utcunque; à quo [a] ducatur HF recta plano ABC, quæ [b] in rectam GK cadet, & quidem [c] perpendiculariter, puta in F. per F ducatur DE ad BC parallela; [d] estque planum per DE, HF parallelum basi BC; [e] efficitque sectionem DHE circulum, in quo FD × FE [f] = FHq. Quia verò ang. ADE [g] = (ang. ABC [h] =) ang. AKG, & ang. KFE [k] = GFD; æquiangula erunt trigona KFE, DFG. unde EF. FK [l] :: GF. FD. ergo FK × GF [m] = (EF × FD [n] =) HFq. [o] quare sectio GHK est circulus. Q.E.D.

a 11. 11.
b 38. 11.
c 3. def. 11.
d 15. 11.
e 4. hujus.
f 35. 3.
g 29. 1.
h hyp.
k 15. 1.
l 4. 6.
m 16. 6.
n prius. o conv. 35. 3.

Prop. VI.

Si conus plano (ABC) per axem secetur, sumatur autem aliquod punctum (D) in superficie coni, quod non sit in latere trianguli per axem; & ab ipso ducatur recta linea (DE) æquidistans cuidam rectæ (MN), quæ perpendicularis est à circumferentia circuli (BMC) ad trianguli basin (BC); triangulo per axem occurret, & ulteriùs producta usque ad alteram superficiei partem, bifariam ab ipso triangulo secabitur. Fig. 10.

Recta AD protracta circumferentiæ basis occurrat puncto K; per quod ducatur KHL ad MN parallela; [a] adeoque ad DE. [b] ergo DE producta occurret rectæ AH, puta in F. Producatur DF ad superficiem G.

a 30. 1. b

Liquet rectam AL, (cùm sit in plano ADE, vel AKL, & in superficie coni) ipsi DF occurrere in G. & fore KH. DF :: (AH. AF ::) HL. FG. unde quum sit KH = HL, erit DF = FG. Q.E.D.

M.

Prop.

Propoſ. VII.

Fig. 11. Si conus plano (A B C) per axem ſecetur, ſecetur autem & altero plano (D F E) ſecanti planum baſis coni, ſecundùm rectam lineam (D E) quæ ſit perpendicularis, vel ad (B C) baſim trianguli per axem, vel ad eam, quæ in directum ipſi conſtituitur: lineæ (H K) quæ à ſectione (D F E), in ſuperficie coni à plano facta, ducuntur æquidiſtantes ei (D E), quæ eſt perpendicularis ad trianguli baſim (B C), in communem ſectionem (F G) plani ſecantis, & trianguli per axem, cadent. Et ſiquidem conus ſit rectus linea (D E), quæ eſt in baſi, perpendicularis erit ad (F G) communem ſectionem plani ſecantis, & trianguli per axem, ſi verò Scalenus, non ſemper, niſi cùm planum (A B C), quod per axem ducitur ad baſim coni (B D C E) rectum fuerit.

Quòd H K plano A B C, & proinde ejus cum plano D F E communi ſectioni F G occurrat, inque ipſo occurſu biſecetur, liquet ex præcedenti. Porro, ſi conus rectus ſit, erit circulus B C plano A B C
a 4. *def.* 11. rectus; [a] ac ideo D E plano A B C recta; & propterea D E ad F G perpendicularis. Idem diſcurſus valet, ſi trigonum A B C circulo B C utcunque rectum fuerit, ſin hoc non fuerit, non erit D E ad F G per-
b 4. 11. pendicularis: Nam ſi D E utrique B C, F G perpendicularis ſit, [b] erit
d 18. 11. eadem D E recta plano A B C; [d] unde circulus B C trigono A B C rectus erit, contra Hypoth.

Coroll. Hinc F G diameter eſt ſectionis D F E, utpote quæ rectas ad D E parallelas biſecat.

Prop. VIII.

Fig. 12. Si conus ſecetur plano (A B C) per axem, & ſecetur altero plano (D F E) ſecanti baſim coni (B D C) ſecundùm rectam lineam (D E), quæ ad (B C) baſim trianguli per axem ſit perpendicularis; diameter autem (F G) ſectionis factæ in ſuperficie, vel æquidiſtet uni (A C) laterum trianguli, vel cum ipſo extra coni verticem conveniat; & producantur in infinitum tum ſuperficies coni (A B C) tum planum ſecans (D F E); ſectio quoque ipſa (D F E) in infinitum augebitur; & ex diametro (F G) ſectionis ad verticem cuilibet lineæ datæ æqualem F G M abſcindet linea (M N H), quæ quidem à coni ſectione (M F N) ei (D E) quæ eſt in baſi, æquidiſtans ducta fuerit.

Nam

Nam quia diameter F G cum latere A C ad partes X [a] nunquam conveniet, si ipsa ad [b] libitum producatur, puta ad H, & per H ducantur K L ad B C, & M N ad D E parallelæ, [c] planum per K L, M N plano B D C E parallelum erit, inque superficie coni producta [d] circulum efficiet, ad quem si protrahatur planum D F E, liquet [d] augeri conum, & sectionem D F E &c. a *Hypoth.* b 3. 1. c 15. 11. d 4. *hujus.*

Prop. IX.

Si conus (A B C) secetur plano (D K E) convenienti cum utroque latere (A B, A C) trianguli per axem, quod neque basi (B C) æquidistet, nec subcontrariè ponatur, sectio (D K E) circulus non erit. Fig. 13. 14.

Si fieri potest, sit D K E circulus, & ab H basis centro ad F G (communem sectionem basis cum plano secanti) ducatur perpendicularis H G, per quam & axem transeat trigonum A B C. Dein sumatur quodvis punctum K in linea D K E, per quod ducatur K M L ad F G parallela, occurrens rectæ D E G (communi sectioni plani secantis, & trigoni A B C) in M. [a] unde K M = M L.

Porrò, per M ducatur N X ad B G parallela. & quia planum per N X, K L, [b] plano B C parallelum est, [*] ideóque circulum efficit. In quo K L [b] diametro N X [c] est perpendicularis, erit N M × M X [d] = (K Mq (d=) D M × M E. ([e]ob D K E circulum). [f] ergo N M. M X :: M E. M X. [g] ergo trigona N M D. E M X similia sunt; & ang. M E X = (ang. D N M [h] =) ang. A B C. Itaque sectio est [i] subcontraria, contra Hyp. ergo sectio D K E non est circulus. *Q. E. D.* a *6. hujus.* b 15. 11. * 14. *hujus.* c 10. 11. (6. d *cor.* 13. & 16. e *hyp.* f 17. 6. g 6. 6. h 29. 1. k 5. *hujus.*

Prop. X.

Si in coni sectione (F E D) sumantur duo puncta (G, H); recta linea (G H), quæ ejusmodi puncta conjungit, intra sectionem cadet, & quæ in directum ipsi constituitur, cadet extra. Fig. 15.

Nam quia puncta G H [a] sunt extra latus trigoni (A B C) per axem ducti, recta G H non pertinget ad verticem A; [b] ergo hæc intra conum cadet, ac proinde intra sectionem; sin producatur extra conum cadet, ac proinde extra sectionem. a *hyp.* b 2. *hujus.*

Prop. XI.

Si conus plano (A B C) per axem secetur, secetur autem & altero plano Fig. 16.

plano (D F E), secante basim coni secundùm rectam lineam (D E), quæ ad basim (B C) trianguli per axem sit perpendicularis, & sit sectionis diameter (F G) uni (A C) laterum trianguli per axem æquidistans; recta linea (K L) quæ à sectione ducitur æquidistans sectioni (D E) plani secantis, & basis coni, usque ad sectionis diametrum (F G), poterit spatium æquale contento, lineâ (F L), quæ ex diametro abscissa inter ipsam (K L) & sectionis verticem (F) interjicitur, & alia quadam (F H) quæ ad lineam (A F) inter coni angulum (A) & sectionis verticem (F) interjectam, eam proportionem habeat, quam quadratum basis (B C) trianguli per axem, ad id quod reliquis duobus trianguli lateribus (A B, A C) continetur. Dicatur autem hujusmodi sectio PARABOLE.

a 15. 11. b 4. *hujus.* c 10. 11. d *cor.* 13. & 16. 6. e 1. 6. f *hypoth.* g 23. 6. h 4. 6. k 2. 6. l 9. 5.

Per L ducatur M N ad B C parallela; estque sectio plani per M N, K L ducti (ad B D C E [a] paralleli) [b] circulus; [c] & K L ad M N perpendicularis; [d] unde K Lq = M L × L N. Porro F L × H F. F L × F A [e] :: (H F. F A [f] :: B Cq. A C × A B [g] = B C. A C. [h] (M L. F L) + B C. A B [h] (M L. F M, [k] vel L N. F A) = M L. F L + L N. F A [g] =) M L. × L N. F L × F A. [l] ergo M L × L N ([d] K Lq) = F L × H F. *Q. E. D.*

Propos. XII.

Fig. 17.

Si conus plano (A B C) per axem secetur; secetur autem & altero plano (D F E) secanti basim coni secundùm rectam lineam (D E) quæ ad basim (B C) trianguli per axem sit perpendicularis, & sectionis diameter (F G) producta, cum uno latere (A C) trianguli per axem extra verticem coni conveniat in (H): recta linea (M N), quæ à sectione ducitur æquidistans communi sectioni (D E) plani secantis, & basis coni usque ad sectionis diametrum, poterit spatium (F N X P) adjacens lineæ (F L), ad quam ea (F H), quæ in directum constituitur diametro sectionis, subtenditúrque angulo (F A H) extra triangulum, eandem proportionem habet, quam quadratum lineæ (A K) quæ diametro (F G) æquidistans, ab vertice (A) sectionis usque ad trianguli basim (B C) ducitur, ad rectangulum basis partibus (B K, K C), quæ ab ea fiant, contentum, latitudinem habens lineam (F N), quæ ex diametro abscinditur, inter ipsam (M N), & sectionis verticem (F) interjectam, excedénsque figurâ ([illegible]) simili, & similiter positâ ei, quæ continetur lineâ (H F) extra angulum subtensâ, & ea (F L), juxta quam possunt, quæ ad diametrum (F G) applicantur. Vocetur autem hujusmodi sectio *Hyperbole*.

EXOL

Per

Per N ducatur RS ad BC parallela. Estque FN * HN. EN * NX [a] :: (HN. NX) [b] :: HE. EL [c] :: AKq. BK * KC [d] = AK. BK ([b] FG. GB, [b] vel FN. NR) + AK. KC. ([b] HG. GC. [b] vel HN. NS.) = FN * NR + HN. NS [a] =) FN * HN. NR + NS. ergo FN * NX [e] = (NR * NS [f] =) NMq. Q. E. D.

a 1. 6.
b 4. 6.
c hyp.
d 23. 6.
e 9. 5.
f 4. hujus, & cor. 13, ac 16. 6.

Prop. XII.

Si conus plano (ABC) per axem secetur, & secetur altero plano (ELD) conveniente cum utroque latere trianguli per axem, quod neque basi coni æquidistet, nec subcontrariè ponatur; planum autem, in quo est coni basis (BC), & secans planum conveniant secundùm rectam lineam (FG) quæ sit perpendicularis vel ad basim (BC) trianguli per axem, vel ad eam (BCK), quæ in directum ipsi constituitur; recta linea (LM) quæ à coni sectione ducitur, æquidistans planorum communi sectioni (FG) usque ad sectionis diametrum (ED) poterit spatium (EOXM) adjacens lineæ (EH), ad quam sectionis diameter (ED) eam proportionem habeat, quam quadratum lineæ (AK) diametro (ED) æquidistantis à coni vertice (A) usque ad trianguli basin (BC) ductæ, habet ad rectangulum contentum basis partibus (BK, CK), quæ inter ipsam (AK) & rectas trianguli lineas (AB, AC) interjiciuntur; latitudinem habens lineam (EM), quæ ex diametro (ED) ab ipsa abscinditur ad sectionis verticem (E): deficiensque figurâ (OHNX) * simili, & similiter positâ ei, quæ diametro (ED) & lineâ (EH) juxta quam possunt, continetur. Dicatur autem hujusmodi sectio *Ellipsis*.

Fig. 18.

Per M ducatur PMR ad BC parallela. Estque EM * DM. EM * MX [a] :: (DM. MX [b] :: DE. EH [c] :: AKq. BK * KC [d] = AK. BK ([b] EG. GB, vel [b] EM. MP) + AK. KC. ([b] DG. GC. [b] vel DM. MR) = EM. MP + DM. MR [a] =) EM * DM. MP + MR. ergo EM * MX [e] = (MP * MR [f] =) MLq. Q. E. D.

a 1. 6.
b 4. 6.
c hyp.
d 23. 6.
e 9. 5.
f 4. hujus, & cor. 13. & 16. 6.

Prop. XIV.

Si superficies (BCAXO), quæ sunt ad verticem (A), plano non per verticem secentur, erit in utraque superficierum sectio (DEF, & GHK) quæ vocatur Hyperbole. Et duarum sectionum eadem erit diameter (MEHN), lineæ verò (EP, HR), juxta quas possunt ordinatæ ad diametrum, æquidistantes ei, quæ est in basi coni, inter se æ-

Fig. 19.

quales erunt. Et figuræ tranſverſum latus (E H) utrique commune, quæ ſcilicet inter ſectionum vertices interjicitur. Vocentur autem hujuſmodi ſectiones *Oppoſitæ*.

Quòd utraque ſectio, D E F, G H K, ſit Hyperbole, liquet ex 12ma hujus. Porrò ductâ per A rectâ S A T ad M N parallelâ, eſt a A S. B S :: A T. T O. & A S. S C :: A T. T X. Unde A Sq. B S. × S C (hoc eſt H E. E P) :: A Tq. T O × T X (hoc eſt E H. H R). quare E P = H R.

a 4. 6.
b hyp.
c 9. 5.

Prop. XV.

Fig. 20. Fig. 21.

Si in Ellipſi à puncto (C) quod diametrum (A B) bifariam dividit, ordinatim ducta linea (D C E) ex utraque parte ad ſectionem producatur, & fiat ut producta (D E) ad diametrum (A B), ita diameter (A B) ad aliam lineam (D F): Recta linea (G H) quæ à ſectione ducitur ad productam (D E) diametro (A B) æquidiſtans, poterit ſpatium (D L) adjacens tertiæ proportionali (D F), latitudinem habens lineam, (D H) quæ inter ipſam & ſectionem interjicitur, deficiénſque figurâ (M K) ſimili ei, quæ continetur lineâ (D E) ad quam ducuntur, & eâ (D F) juxta quam poſſunt. Quòd ſi ulterius producatur (G H) ad alteram ſectionis partem (V), bifariam ſecabitur ab ea (D E), ad quam applicata fuerit.

Sit A N linea, juxta quam poſſunt applicatæ ad A B: junctâque B N, per G ducatur G X ad D E parallela, quæque C & X ipſæ X O, C P ad A N parallelæ; & per N, O, P ipſi A B parallelæ N R, O S, P T. Liquet igitur eſſe D Cq = rectang. A P. & G Xq = rectang. A O. Et ob A N C P. (A T) :: (A B. C B :: 1. 1.) erit T N = A T. unde rectang. A P = N P. & rectang. X T = Y T = N S (ob T O = R O). ergo rectang. A O. (G Xq) + O P = (rectang. N R = A P = C Dq = H Cq (G Xq) + H E × H D. & proinde rectang. H E × H D = rectang. O P. (P S × S O). Item H E × H D. H L × H D :: (H E. H L :: D E. D F :: D Eq. A Bq (ob D E, A B, D F ÷) :: C Dq (P C × C A, vel P C × C B). C Bq :: P C. C B :: P S. S O :: P S × S O. (H E × H D). S Oq. ergo H L × H D q = (S Oq =) H Gq. Quod erat primum. Porrò, ductis V Q ad G X, & Q Z ad A N parallelis, propter A X × X O = (G Xq = V Q q =) A Q × Q Z, erit A Q. A X :: (X O. Q Z ::) X B. Q B. ergo dividendo X Q. X A :: X Q. Q B. quare

a 13. hujus.
b 4. 6.
c hyp.
d 1. 6.
e 2. aux. 1.
f 43. 1.
g 5. 2.
h 3. aux. 1.
k 1. 6.
l 4. 6.
m cor. 20. 6.
n hyp.
o 15. 5.
p 13. 6.
q 9. 5.
r 34. 1.
s 14. 6.

quare X A = Q B. item C A [v] = C B. [x] ergo C X = C Q, [z] hoc [z] 9. 5.
est H G = H V. Quod erat secundum. [v] byp. [x] 3. ax. 1.

Coroll. Itaque D E est diameter altera priori A B conjugata.

Schol.

Brevius ita Calculum instituemus.

Sint $\begin{cases} B C, \text{ vel } C A = d. \\ A N = r. \\ C X, \text{ vel } H G = a. \end{cases}$ Fig. 22.

Est igitur $\begin{cases} B X = d + a. \\ A X = d - a. \end{cases}$

Est autem $2d. r :: d + a. \frac{r}{2} + \frac{ra}{2d}$ quare $d - a\ (A X) \times : \frac{r}{2} + \frac{ra}{2d}$ hoc est $\frac{dr}{2} - \frac{raa}{2d} = G Xq$ vel $H Cq$. Item quia $2d. r\ (B A. AN) :: d.$ $(B C). \frac{r}{2}$. erit $d (B C) \times \frac{r}{2}$, h. e. $\frac{dr}{2} = DCq$. Ergo $DCq - HCq$ (h. e. $HE \times HD) = \frac{raa}{2d}$. Porrò quia $DCq. BCq$ (hoc est $\frac{dr}{2}. dd$) :: $DEq. ABq :: DE. DF$ (ob $DE, AB, DF \frac{\cdot\cdot}{\cdot\cdot}$) :: $HE. HL ::$ $HE \times HD. \left(\frac{raa}{2d}\right). HL \times HD :: \frac{dr}{2}. dd :: \frac{r}{2}. d :: \frac{raa}{2d}. aa$. Erit $HL \times HD = aa = HGq$.

Simili discursu erit $HL \times HD = HVq$. unde $HG = HV$.

Prop. XVI.

Si per punctum (C), quod transversum latus (AB) oppositarum sectionum bifariam dividit, recta quædam linea (C D) ordinatim applicetur, ipsarum diameter erit, priori diametro (A B) conjugata. Fig. 23.

Recta quæpiam G H ad A B parallela sectionibus occurrat punctis G, H, à quibus ordinatim applicentur G K, H L; sintque A E. B F [a] 13. *hujus.* recta sectionum latera; & junctæ A F, B E producantur; ducan- [b] 34. 1. turque K M, L N ad A E, B F parallelæ. Estque $AK \times KM$ [c] = [c] 1. 6. $(GKq$ [b] $= HLq$ [a] $=) BL \times LN$. Item $AK \times KM. AK \times KB$ [c] :: [d] 4. 6. (5. $(KM. KB$ [d] $:: AE. AB$ [e] $:: BF. BA$ [e] $:: LN. LA$ [c] $::) BL \times$ [c] 14. *hujus.* [e] *prius.* $LN. BL \times LA$. ergo (ob $AK \times KM$ [f] $= BL \times LN$) [g] erit $AK \times$ [f] 14. 5. $KB = BL \times LA$. [h] quare $KB. BL :: LA, AK$, & componendo [g] 14. 6. [h] KL.

k 9. 5. KL. BL :: LK. AK. [k] ergo BL = AK. & [l] proinde CL = CK;
l hyp. & 3. ax. [m] hoc est XH = XG. [n] ergo CD est diameter, quippe quæ ipsam
m 34. 1. GH, & similiter omnes ad AB parallelas bisecat.
n 12. def. hujus.

Coroll. 1. Si GK = HL, erit AK = BL; & BK = AL. ac inde BK × AK = AL × BL.

2. Vicissim, si AK = BL, vel BK = AL. erit GK = HL. & BK × AK = AL × BL.

DEFINITIONES SECUNDÆ.

Fig. 24. 25. 1. Punctum (C), quod hyperbolæ, & ellipsis diametrum (AB) bifariam dividit, Centrum sectionis dicatur.

2. Et quæ à centro (C) ad sectionem perducitur (CB) vocetur ex centro sectionis.

3. Similiter & punctum (C) quod transversum latus (AB) oppositarum sectionum bifariam dividit, Centrum vocetur.

4. Quæ autem (DE) à centro ducitur æquidistans ei (GK), quæ ordinatim applicata est, mediámque proportionem habet inter latera figuræ (AB, BF) & bifariam secatur à centro (C), secunda diameter appellatur.

Brevitatis causâ, Transversum latus, Rectum latus, & secundam diametrum elementis T, R, M designabimus: unde

o Cor. 20. 6. *Coroll.* * Tq. Mq :: T. R. (ob T, M, R $\frac{\cdot}{\cdot}$.)
& $C Dq = \frac{1}{4} D Eq = \frac{Mq}{4} = \frac{1}{4} T R.$

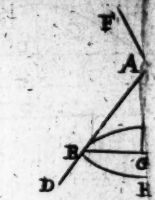
Viam jam munit ad singularum sectionum proprietates primas, & præcipuas eliciendas.

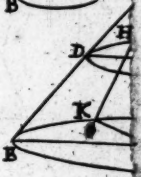
Prop. XVII.

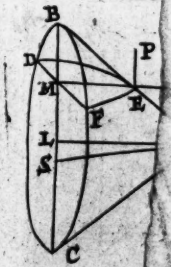
p Fig. 26. Si in conisectione (C A D), ab ipsius vertice (A) ducatur recta linea (A C) æquidistans ei, quæ ordinatim applicata est, extra sectionem cadet.

a 7. hujus. b 10. hujus. Nam si dicatur intra cadere, [a] ergo bisecabitur à diametro; quod fieri nequit, [b] eum producta extra sectionem cadat.

Prop.







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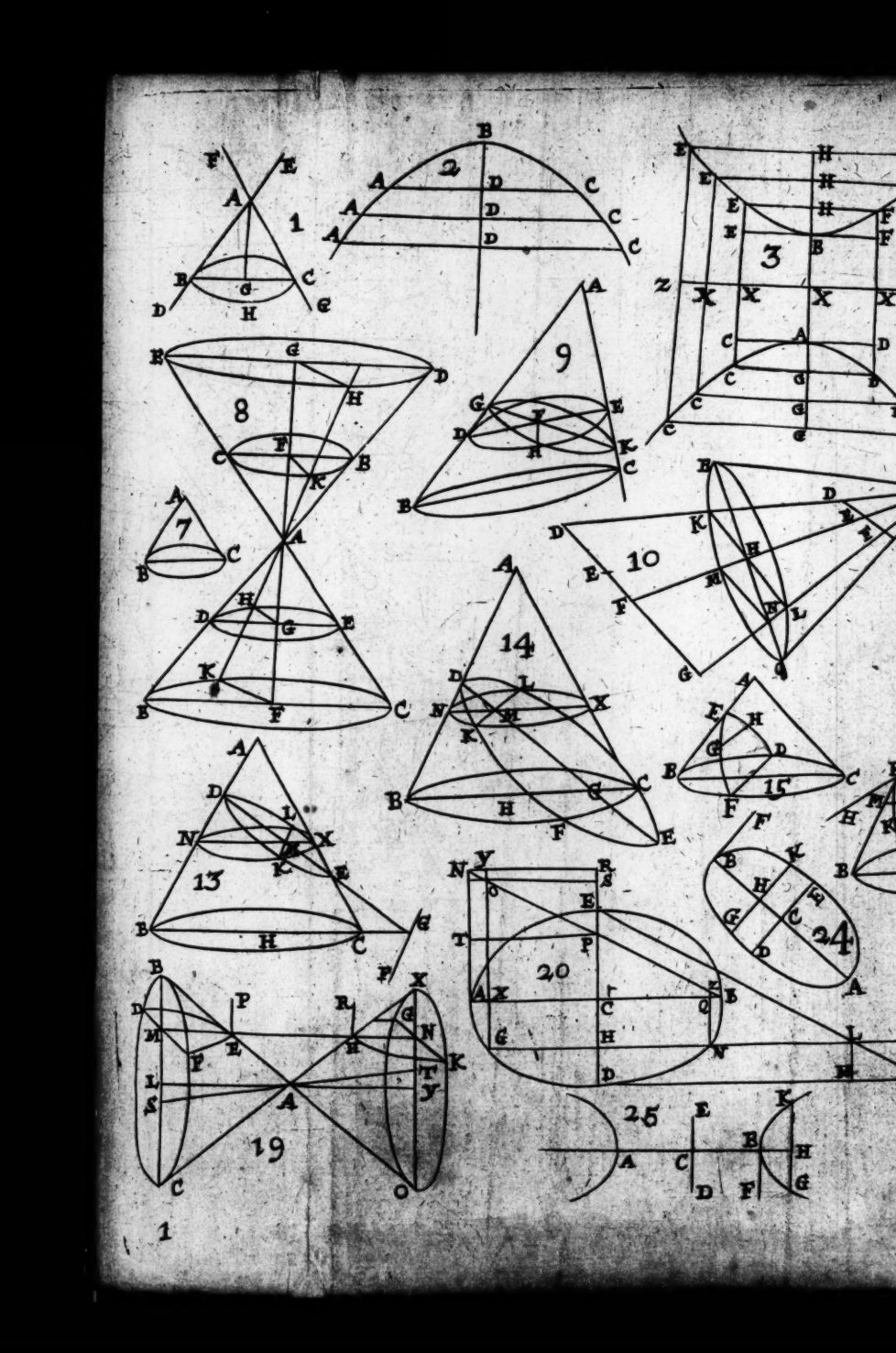
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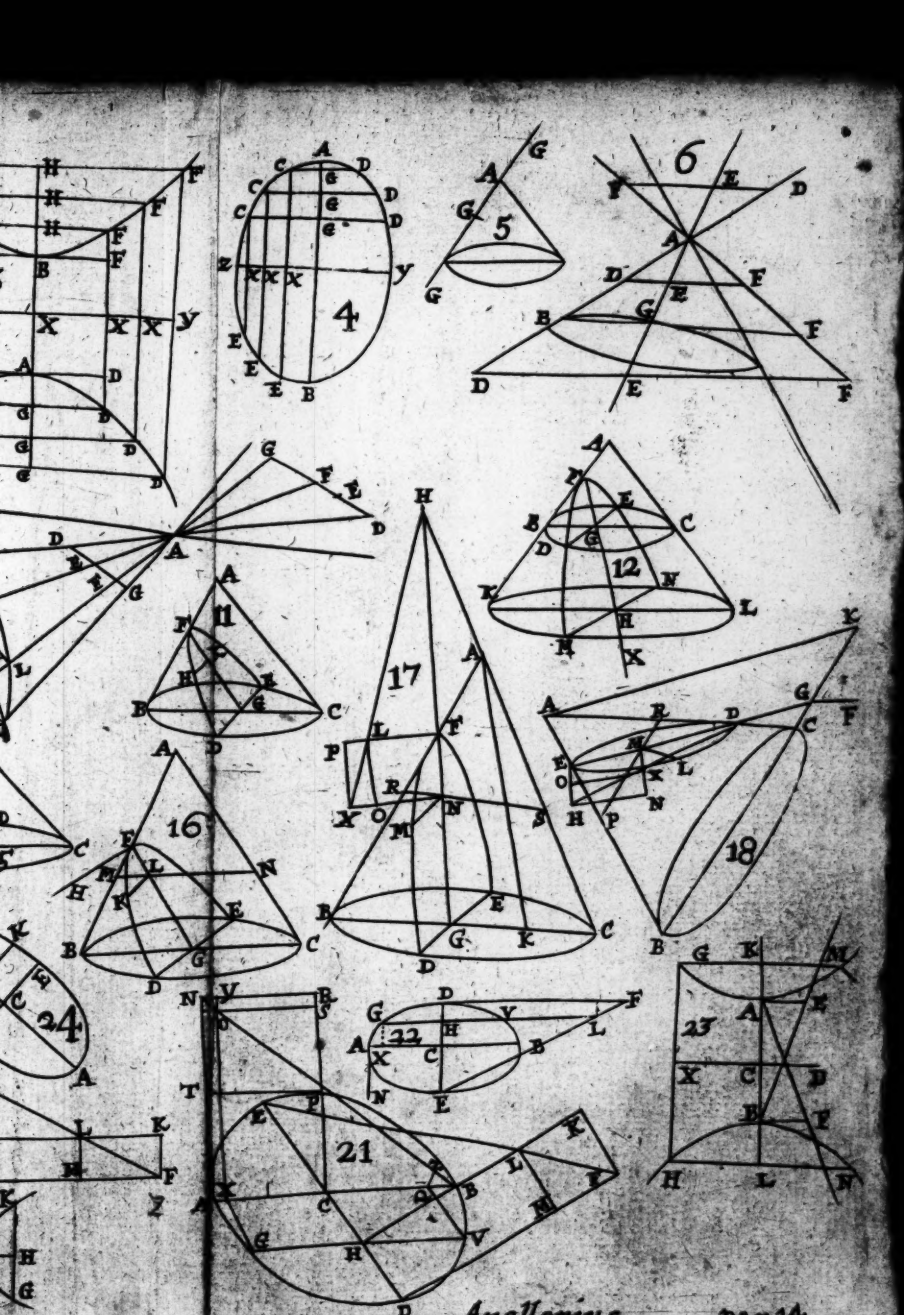
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Prop. XVIII.

Si recta linea (A F B) sectioni occurrens (in F), productáque in utramque partem, extra sectionem cadat; sumatur autem aliquod punctum (C) intra sectionem, & per ipsam ei (A B), quæ sectioni occurrit, ducatur æquidistans (C D); ducta linea (C D), & producta, ex utraque parte sectioni occurret. Fig. 27.

Sumatur in sectione punctum quodvis E, & connectatur E F; hæc ipsi C D occurret; & siquidem inter puncta E F, manifestum est ipsam sectioni occurrere; sin extra, tum prius cum sectione conveniet. Simili discursu ad partes A F sectioni occurret.

Prop. XIX.

In omni sectione coni recta linea (B C) quæ à diametro (A B) ducitur, ordinatim applicatæ æquidistans, cum sectione conveniet. Fig. 28.

Sumatur aliquod punctum D in sectione, jungatúrque A D; hæc occurret ordinatim applicatæ ad A, ergo ad illam parallelæ A C; & siquidem inter puncta A D, tum B C [a] protracta sectioni occurret, sin extra, prius. [a] 10. *hujus*.

prop. XX.

Si in Parabola duæ rectæ lineæ C E, D F à sectione ad diametrum (A B) ordinatim applicentur, ut earum quadrata inter se, ita erunt lineæ (A E, A F) quæ ab ipsis ex diametro ad verticem abscinduntur. Fig. 29.

Sit A G latus rectum. itaque C Eq [a] = A E * A G. [a] & D Fq = A F * A G. ergo C Eq. D Fq [b] :: (A E * A G. A F * A G [c] ::) A E. A F. *Q. E. D.* [a] 11. *hujus*. [b] 7. 5. [c] 3. 6.

Converſim. Si sit C Eq. D Fq :: A E. A F, erunt puncta C, D in parabola.

Coroll. D F ⊏ C E.

Hæc prima & præcipua est parabolæ proprietas, ex ejus definitione emergens.

Prop. XXI.

Si in hyperbola, vel ellipsi vel circuli circumferentia, rectæ lineæ (D E, F G) ordinatim ad diametrum (A B) applicentur, erunt quadrata. Fig. 30.

drata earum ad ſpatia contenta lineis (E B, E A, & G B, G A) quæ inter ipſas, & vertices (A, B) tranſverſi lateris figuræ interjiciuntur, ut figuræ rectum latus (A C) ad tranſverſum (A B); inter ſe verò ut ſpatia, quæ interjectis, ut diximus, lineis continentur.

Per E, & G ducantur E H, G K ad A C parallelæ, occurrentes productæ B C in H, & K. Eſtque C A. A B [a] :: (H E. E B [b] ::) H E × E A ([c] hoc eſt D Eq). E B × E A. Simili diſcurſu C A. A B :: F Gq. G B × G A. Itaque D Eq. E B × E A [d] :: F Gq. G B × G A. & viciſſim D Eq. F Gq :: E B × E A. G B × G A.

a 4. 6.
b 1. 6.
c 12. *hujus.*
d 11. 5.

Coroll. In hyperb. F E ⊏ D E. (in ellipſi uſque ad conjugatam ipſi A B diametrum, nam inde incipiunt ordinatim applicatæ decreſcere.) *Sch.*

Converſim; ſi fuerit D Eq. E B × E A :: R T. vel D Eq. F Gq :: E B × E A. G B × G A. erunt puncta D, F in aliqua harum ſectionum.

Hæc prima eſt & præcipua harum ſectionum proprietas, ex ipſarum definitione reſultans.

Coroll. D Eq. E B × B A :: Tq. Mq.

Viam jam ſternit ad ſectionum tangentes indagandas.

Prop. XXII.

Si Parabolen, vel hyperbolen recta linea (C D) in duobus punctis (C, D) ſecet, non conveniens cum ſectionis diametro (A B) intra ſectionem, producta cum eâdem diametro extra ſectionem convenjet. Fig. 31. 32.

Ordinatim applicentur C E, D B: ſunt hæ inæquales, & [a] minor D B, [b] ergo juncta C D producta cum E A conveniet extra ſectionem ad partes A. *Q. E. D.*

a *cor.* 20. &
b *cor.* 21. *hujus.*

Prop. XXIII.

Si ellipſin ſecet recta linea (E F) inter duas *diametros (A B, C D) producta, producta cum utraque earum convenjet, extra ſectionem.

**id eſt*, conjugatas.

Ordinatim applicentur E G, F H, atque ob E Gq. F Hq [a] :: B G × G A. B H × H A, [b] & B G × G A ⊏ B H × H A (eſt enim punctum G propius centro M, quam punctum H); [c] erit E Gq ⊏ F Hq. & E G ⊏ F H. ergo E F producta cum diametro B A conveniet, ad partes Fig. 33.

a 21. *hujus.*
b 5. 2.
c 14. 5.

partes A. Simili discursu eadem F E diametro D C occurret ad partes C.

Coroll. E G ⊏ F H.

Prop. XXIV.

Si parabolæ, vel hyperbolæ recta linea (C E) in uno puncto (D) occurrens, & producta ex utrâque parte, extra sectionem cadat, cum diametro (A B) conveniet. Fig. 34.

Sumpto quolibet in sectione puncto F, jungatur F D ; [a] hæc diametro occurret ; puta in A ; hanc verò decussat ipsa C D (in D). ergo C D producta diametro occurret ; inter A scilicet & sectionem. [a] 22. *hujus.*

Prop. XXV.

Si ellipsi recta linea (E F) occurrens (in G), inter duas diametros (A B, C D) & producta ex utrâque parte cadat extra sectionem, cum utrâque diametro conveniet. Fig. 35.

Ordinatim applicetur G K ; hæc diametro A B est parallela : ergo E F cum A B conveniet. Simili discursu, F E cum C D conveniet.

Prop. XXVI.

Si in parabola, vel hyperbola ducatur recta linea (C D) sectionis diametro (A B) æquidistans, in uno tantùm puncto (E) cum sectione conveniet. Fig. 36.

Quod conveniet C D cum sectione, patet, (quoniam distantia parallelarum C D, A B est finita, sectio autem in infinitum potest augeri ; adeóque ducta aliqua ab A B ordinatim applicata ad sectionem, excedet istam distantiam.) Conveniat in E ; & ordinatim applicetur E F : ergo cùm omnes ordinatim applicatæ ad partes D [a] majores sint quàm E F, ad partes verò C minores, liquet C D nusquam convenire cum sectione, præterquam in E. [a] *cor.* 20. & *cor.* 21. *hujus.*

Prop. XXVII.

Si parabolæ diametrum (A B) secet recta linea (C D), producta in utramque partem cum sectione conveniet. Fig. 37.

Sit A E ordinatim applicatis parallela ; si C D huic parallela sit,

[a] liquet ipſam utrinque ſectioni occurrere, ſin minùs, producta conveniet cum A E, puta in E, [b] ergò priùs cum ſectione, puta in G; ordinatim igitur applicetur G F; [c] fiátque A F. A D :: AD. AB. [d] unde F D. D B :: A D. A B. ductâ igitur B C ad G F parallela erit, F Dq. D Bq. ([e] hoc eſt G Fq. B Cq) [f] :: A Dq. A Bq [g] :: A F. A B. ergo cùm ſit G Fq. B Cq :: A F. A B, [h] erit punctum C in ſectione; quare C D utrinque ſectioni occurrit.

a 19. *hujus.*
b 17. *hujus.*
c 11. 6.
d 19. 5.
e 4. & 22. 6.
f 22. 6.
g *conſt. & cor.* 20. 6.
h *conv.* 20. *hujus.*

Prop. XXVIII.

Fig. 38. Si recta linea (C D) unam (A) oppoſitarum ſectionum contingat, ſumatur autem punctum (E) intra alteram ſectionem (B), & per ipſum linea (E F) contingenti æquidiſtans ducatur, producta ad utramque partem cum ſectione (B) conveniet.

Nam quia C D diametro occurrit, eidem occurret E F, puta in G. Sit A H = B G, & per H ducatur H K ad C D vel E F parallela, ſectioni occurrens in K, & ordinatim applicetur K L, ſumatúrque G M = H L; denique ducatur M N ad K L parallela. Eſtque H L. L K :: [a] G M. M N. ergo (ob H L [b] = G M) [c] erit L K = M N. [d] Item B L × A L = A M × B M. quare B L × A L. L Kq [e] :: A M × B M. M Nq. [f] unde punctum N erit in ſectione B. Simili argumento ex altera parte occurret ipſa E F ſectioni.

a 4. 6.
b *conſtr.*
c 14. 5.
d *conſtr. & Sch.* 48. 1.
e 7. 5.
f 21. *hujus.*

Prop. XXIX.

Fig. 39. Si in oppoſitis ſectionibus recta linea (C D) per centrum (C) ducta occurrat uni ſectioni (A), ulteriùs producta alteram quoque (B) ſecabit.

Ad diametrum A B ordinatim applicetur D F, fiátque B G = A F, & ordinatim ducatur G E, jungatúrque C E. éſtque B F × A F. D Fq [a] :: T. R [a] :: A G × B G. G Eq. ergo cùm [b] ſit B F × A F = A G × B G. [c] erit D Fq = G Eq. & D F = G E. [d] Item C F = C G. & ang. F [e] = ang. G. [f] ergo ang. F C D = G C E: [g] quare linea D C E eſt una recta, ſectioni B occurrens in E.

a 21. *hujus.*
b *conſtr. & ſch.* 48. 1.
c 14. 5.
d *conſtr.*
e 29. 1.
f 4. 1.
g *Sch.* 15. 1.

Coroll. 1. D F = E G.
2. C D = C E; (ob trigona C F D, C G E ſimilia, & latera C F, C G æqualia.)

Prop.

Prop. XXX.

Si in ellipſi vel oppoſitis ſectionibus ducatur recta linea D E, ad utraſque partes centri (C) ſectioni occurrens, ad centrum (C) bifariam ſecabit. Fig. 40.

In oppoſitis ſectionibus patet * ex præcedenti. In ellipſi verò ad diametrum A B ordinatim applicentur D F, E G. Et quoniam B F × F A. A G × G B [a] :: (D Fq. G Eq [b] ::) F Cq. G Cq. & permutando B F × F A. F Cq :: A G × G B. G Cq. & componendo A Cq. (B F [c] × F A + C Fq). F Cq :: B Cq. [c] (A G × G B + G Cq) G Cq. ſitque A C [d] = B C. [e] erit F C = G C; & propterea C D [f] = C E.

* cor. 2. præc.
a 21. hujus.
b 4. 6. & 22. 6.
c 5. 2.
d conſtr. & 10. def. hujus.
e 14. 5.
f 4. 6.

Coroll. C G = C F, & G E [g] = D F; & G B [h] = F A. & B F = A G. & B F × F A [h] = A G × G B.

g 3. ax. 1.
h Sch. 48. 1.

prop. XXXI.

Si in transverſo figuræ latere (A B) hyperbolæ, ſumatur aliquod punctum (C) non minorem (C B) abſcindens ad verticem ſectionis, quàm ſit dimidia tranſverſi lateris (A B) figuræ, & ab ipſo (C) recta linea (C D) ſectioni occurrat, ſi producatur, intra ſectionem ad ſequentes ipſius partes (E) cadet. Fig. 41.

Sit primò A C = C B. & ordinatim applicentur D H, F E G. Et quia C Gq. C Bq [a] ⊏ C Hq. C Bq. erit per converſam rationem C Gq. C Gq — C Bq ⊐ C Hq. C Hq — C Bq. & inverſè C Gq — C Bq. C Gq ⊏ C Hq — C Bq. C Hq. & permutatim, C Gq — C Bq. [b] (A G × G B.) C Hq — C Bq [b] (A H × H B) ⊏ (C Gq. C Hq [c] ::) G Eq. H Dq). ergo cum A G × G B. A H × H B [d] :: G Fq. H Dq. erit G Fq. H Dq [e] ⊏ G Eq. H Dq. [f] ergo G F ⊏ G E. ergo C D E intra ſectionem cadit.

* utcunque hæc ad partes E.
a 8. 5.
b 6. 2.
c 4. & 22. 6.
d 21 hujus.
e 13. 5.
f 10. 5.

Quòd ſi ab aliquo puncto in A C (poſito C centro) ad punctum D ducatur recta, hæc ipſam C D decuſſabit in D, adeóque magis intra ſectionem cadet.

Coroll. Hinc, linea hyperbolem contingens, diametrum ſecat inter verticem, & centrum ſectionis: unde multò magis

Linea quæ duobus punctis ſecat (vel quæ tangenti parallela eſſe poterit) diametro occurret inter verticem & centrum.

Prop. XXXII.

Fig. 42. Si per coni ſectionis verticem (A) ducatur recta linea (A C) ordinatim applicatis æquidiſtans, ſectionem continget, & in locum (CAG), qui inter coni ſectionem & rectam lineam (A C) interjicitur, altera recta linea non cadat.

Fig. 43. 44. Si fieri poteſt, cadat A D, & à puncto D (utcunque ſumpto in A D) ordinatim applicetur D G E, ſintque A F, B A latera figurarum. Jam in parabola fiat A F. A H :: D Eq. A Eq. & ducatur H K ad E D parallela, ſectioni occurrens in L. Eſtque A F × A H [a] (L Hq). A Hq [b] :: (A F. A H [c] :: D Eq. A Eq. [d] ::) K Hq. A Hq. [e] ergo L H = K H. [f] *Q. E. A.*

a 11. *hujus.* b 1. 6. c *conſtr.* d 4. & 22. 6. e 9. 5. f 9. *ax.* 1.

In reliquis ſectionibus, præter hæc, connexa B F producatur; & per E ducatur E M N ad A F parallela, fiátque A E × E N = D Eq; & juncta A N ſecet B [illegible] in X; ducantur X H ad A F, & H K ad A C parallelæ. Eſtque X H × A H [g] (L Hq). A Hq [b] :: (X H. A H [h] :: N E. E A [b] :: N E × E A [g] (D Eq). E Aq [d] ::) K Hq. A Hq. ergo L Hq [f] = K Hq. & L H = K H, pars toti æqualis. [f] *Q. E. A.*

g 12. *hujus.* h 4. 6.

Cor. Si duæ ſectiones ſeſe contingant, quæ harum unam contingit recta, alteram quoque continget.

Prop. XXXIII.

Fig. 45. Si in parabola, ſumatur aliquod punctum (C), à quo recta linea (C D) ad diametrum (A B) ordinatim applicetur; & ei (E D) quæ ab ipſa ex diametro abſcinditur ad verticem, æqualis (A E) ponatur in directum ab ejus extremitate (E); recta linea (A C), quæ à facto puncto (A) ducitur ad illud (C) quod ſumptum fuerat, ſectionem continget.

Sumpto utcunque puncto F in A C, ordinatim applicetur F B, ſectioni occurrens in G. Et quoniam 2 A E × E B [a] ⊐ A Eq + E Bq; erit 4 A E × E B [b] ⊐ (A Eq + E Bq + 2 A E × E B [c] =) A Bq; [c] atqui 4 A E × E D. = A Dq. ergo 4 A E × E B. A Bq [d] ⊐ 4 A E × E D. A Dq. & permutando 4 A E × E B. 4 A E × E D ([e] hoc eſt, E B. E D, [f] vel G Bq. C D q) ⊐ A Bq. A Dq. ([g] vel F Bq. C D q) [h] ergo G B. C D ⊐ F B. C D. [k] quare G B ⊐ F B. unde punctum F eſt extra ſectionem; idemque de reliquis rectæ

a *Not. infra.* b 4. *ax.* 1. c 4. 6. d 13. 5. e 1. 6. f 20. *hujus.* g 4. & 22. 6. h 22. 6. k 10. 5.

rectæ AC punctis ſimili diſcurſu oſtendetur. ergo recta ACF ſectionem contingit. *Q. E. D.*

Not. $2AE \times EB \sqsupset AEq + EBq$. Nam $AEq + EBq = 2AE \times EB +$ Quad: $AE - EB$ (z hoc eſt $+ AEq + EBq - 2AE \times EB$). z *cor.* 7. 2.

Prop. XXXIV.

Si in hyperbola, vel ellipſi, vel circuli circumferentia ſumatur aliquod punctum (C), ab eóque recta linea (CD) ad diametrum (AB) ordinatim applicetur, & quam proportionem habent lineæ (BD, AD), interjectæ inter ordinatim applicatam (CD) & (A, B) terminos tranſverſi lateris (AB) figuræ, eandem habeant inter ſe (BE, EA) partes lateris tranſverſi, ita ut quæ ſunt ad verticem partes ſibi ipſis reſpondeant (BD. DA :: BE. EA), recta linea (EC) conjungens punctum (E) quod in tranſverſo latere ſumitur, & punctum (C) quod eſt in ſectione, ſectionem ipſam continget. Fig. 46. 47.

Sumpto utcunque puncto F in EF ordinatim applicetur FG, ſectioni occurrens in H. & per A, B puncta ducantur AL, BK ad EF parallelæ, & protrahantur DCK, BCX, GCM. Eſtque BK. AN [a] :: (BD. DA [b] :: BE. EA [a] :: BC. CX [a] ::) BK. NX. [c] ergo $NX = AN$. quare $NX \times AN$ [d] $\sqsubset AO \times OX$. [e] ideóque NX. OX [f] (hoc eſt KB. MB) [g] $\sqsubset AO. AN$ unde $KB \times AN \sqsubset MB \times AO$. [h] ergo $KB \times AN$. CEq ([i] hoc eſt BD × DA. DEq.) $\sqsubset$ BM × AO. CEq [k] (hoc eſt BG × GA. GEq.) & permutando BD × DA. BG × GA [l] (hoc eſt CDq. HGq) $\sqsubset$ DEq. GEq. [m] (CDq. FGq) [n] ergo CD. HG $\sqsubset$ CD. FG. [o] ergo $HG \sqsupset FG$. quare punctum F extra ſectionem exiſtit. Quare EF ſectionem contingit.

a 4. 6. b *hyp.* c 9. 5. d 5. 2. e *Not. 2. infra.* f *Not. 1.* g 8. 5. h 3. *Not.* k 21. *hujus.* l 4, & 22. 6. m 22. 6. n 10. 5.

Notæ.

1. Quod ſit NX. OX :: KB. MB, ſic patet: quoniam NO. KM [o] :: (OC. CM [o] ::) OX. MB, & permutatim NO. OX :: KM. MB. erit componendo NX. OX :: KB. MB. o 4. 6.

2. Quod ſit $NX. OX \sqsubset AO, AN$, ſic oſtenditur. Sit $RS \sqsubset XY$. Dico R. X $\sqsubset$ Y. S. Sit enim $RS = XZ$. [p] ergo $Z \sqsubset Y$. [q] ergo Z. S [r] (hoc eſt R. X) $\sqsubset$ Y. S. p *ſch.* 48. 1. q 8. 5. r 17. 6.

3. Quod ſit KB × AN. CEq :: BD × DA. DEq, ita conſtabit: quoniam AN. CE [s] :: AD. DE. & CE. KB [s] :: DE. DB erit ex s 4. 6.

ex quo

t 1. 6. u 4. & 22. 6. æquo AN. KB t (ANq. AN × KB) :: AD. DB t (ADq. AD × DB). u Item C Eq. ANq :: D Eq. ADq. ergo rursus ex æquo C Eq. AN × KB :: D Eq. AD × DB, ac inversè.

Coroll.

Hinc si $\frac{T \times AD}{T + 2AD} = AE$ in hyperbola, vel $\frac{T \times AD}{T - 2AD}$ in ellipse, erit EC tangens.

Prop. XXXV.

Fig. 48. Si parabolen recta linea (AC) contingat, conveniens cum diametro (AB) extra sectionem (in A), quæ (CB) à tactu ad diametrum ordinatim applicatur, abscindet ex diametro ad verticem sectionis lineam (BG) æqualem ei (GA), quæ inter ipsam & contingentem interjicitur; & inter locum, qui est inter contingentem, & sectionem, alia recta linea non cadet.

Si fieri potest, sint AG, GB inæquales; ipsique AG æqualis ponatur GE; & ordinatim applicetur EF. a ergo ducta AF sectionem a 33. hujus. b 14. ax. 1. continget, & rursus occurret ipsi AC. b Q. E. A.

Porro dic aliquam DC sectionem inter & AC cadere; fiátque GE = GD. & ordinatim applicetur EF. a ergo ducta DF tanget sectionem, ipsámque DC iterum decussabit. b Q. E. A.

Prop. XXXVI.

Fig. 49. 50. Si hyperbolén vel ellipsin, vel circuli circumferentiam contingat quædam recta linea (CD) conveniens cum transverso figuræ latere (BA), & à tactu recta linea (CE) ad diametrum (AB) ordinatim applicetur; erit ut linea (BD) quæ interjicitur inter contingentem, & terminum (B) transversi lateris ad (DA) interjectam inter eandem, & alterum lateris terminum (A), ita linea (BE), quæ est inter ordinatim applicatam (CE) & lateris terminum (B) ad eam (EA), quæ est inter eandem (CE) & alterum terminum (A), adeò ut continuatæ inter se sint, quæ sibi ipsis respondent (BD. DA :: BE. EA). Et in locum, qui inter contingentem, & sectionem coni interjicitur, altera recta linea non cadet.

Si enim non sit BD. DA :: BE. EA; sit BD. DA :: DG. GA. a 34. hujus. b 14. ax. 1. & ordinatim applicetur GF; a ergo ducta DF sectionem continget, iterúmque conveniet cum recta b DC. Q. E. A.

Quin-

Quinetiam si aliqua H C intercedat, fiat B H. H A :: B G. G A. & applicetur G F ordinatim : [a] itaque juncta H F sectionem continget, [b] ipsamq; D G bis decussabit. *Q. E. A.*

Cor. Hinc si C D tangat, erit in hyperbola $AE = \frac{T \times AD}{T - 2AD}$; in ellipse $AE = \frac{T \times AD}{T + 2AD}$.

Nota. In hyperbola $AE \subset AD$: quia $T - 2AD \dashv T$.
In ellipse $AE \supset AD$, quia $T + 2AD \subset T$.

Prop. XXXVII.

Si hyperbolen vel ellipsim, vel circuli circumferentiam recta linea (C D) contingens cum diametro (A B) conveniat, & à tactu (C) ad diametrum linea (C E) ordinatim applicetur; quæ (E F) interjicitur inter applicatam (C E) & sectionis centrum (F), una cum interjecta (D F) inter contingentem (C D) & sectionis centrum (F); continebit rectangulum æquale quadrato lineæ F B, quæ est ex centro sectionis; sed una cum ea (E D) quæ inter applicatam & contingentem interjicitur, continebit spatium, quod ad quadratum lineæ applicatæ (C E), eandem proportionem habet, quam transversum figuræ latus ad rectum. Fig. 51. 52.

Nam A E. E B [a] :: A D. D B ergo componendo A E + E B. E B :: A D + D B. D B quare (in hyperbola) bipartiendo antecedentes, F E. E B :: F B. D B. & per conversam rationem, F E. F B :: F B. F D. [b] unde F E × F D = F Bq. *Q. E. D.*

Item, inverse F B [c] (A F) F E :: (F D. F B [d] ::) D B. (F B — F D). E B (F E — F B). ergo componendo. A E. F E :: D E. E B. ergo F E × D E [e] = A E × E B. quare F E × D E. C Eq [f] :: (A E × E B. C Eq [g] ::) T. R. *Q. E. D.*

In Ellipsi verò & circulo, ob A E + E B. E B :: A D + D B. D B. erit quoque (bipartiendo antecedentes,) F B. E B :: F D. D B. & per conversam rationem F B. F E :: F D. F B. unde F E × F D [h] = F Bq. hoc est D E × F E + F Eq [i] = (F Bq =) A E × E B + F Eq [k] ergo D E × F E = A E × E B. & D E × F E. C Eq [l] :: (A E × E B. C Eq [m] ::) T. R.

a 36. *hujus.*
b 17. 6.
c *hyp.*
d 19. 5.
e 16. 6.
f 7. 5.
g 21. *hujus.*
h 17. 6.
k 3. 2.
l 5. 2.
m 3. *ax.* 1.
n 7. 5.
o 21. *hujus.*

Coroll. F E, F B, F D sunt ∴.
F E. E B :: F B. D B.
F B. B E :: F D. D B.
F B. F D :: B E. D B.

Hinc

Hinc methodus ex dato puncto in diametro, vel sectione contingentem ducendi.

Conversim: si FE × FD = FBq: vel si DE × FE. CEq :: T,R. erit recta CD tangens sectionis.

Prop. XXXVIII.

Fig. 53. 54. 55. 56. Si hyperbolen, vel ellipsin, vel circuli circumferentiam recta linea (FL) contingens in (E), cum secundâ diametro (CD) conveniat, & à tactu (E) ad diametrum (CD) applicetur linea (EH) æquidistans alteri diametro (AB); quæ (HG) interjicitur inter applicatam (EH) & sectionis centrum (G) unà cum interjecta (FG) inter contingentem, & sectionis centrum, continebit rectangulum, æquale quadrato quod fit ex (GC) dimidia secundæ diametri: sed unà cum ea (HF) quæ inter applicatam & contingentem interjicitur, spatium continebit, quod ad quadratum applicatæ (EH) eam proportionem habet, quam figuræ rectum latus ad transversum.

Ordinatim applicetur EM. Estque GM × GL. HG × FG [a] = GM. HG + GL. FG [b] = GM. EM + LM. EM [a] = GM × LM. EMq [c] = T. R. [d] :: Tq. Mq ([e] ABq. CDq [d] ::) AGq. CGq. ergo cum AGq [c] = GM × GL [f]. erit CGq = HG × FG. *Q.E.D.*

Porro, inversè R. T = (HG. GM + FG. GL [b] = HG. HE + FH. HE [a] =) HG × FH. HEq.

Iisdem positis ostendendum est. Ut linea (CF), quæ inter tangentem, & terminum secundæ diametri ad partes applicatæ interjicitur, ad eam (FD), quæ inter tangentem, & alterum terminum secundæ diametri, ita esse lineam (DH), quæ est inter alterum terminum, & applicatam, ad eam (CH) quæ inter alterum terminum & applicatam.

Nam ob FG × HG [g] = (CGq [h] =) CG × GD, [k] erit FG. GD :: CG. GH. & per conversam rationem GF. FD :: GC. CH. & duplicando antecedentes, CF + FD. FD :: DC. CH. & dividendo CF. FD :: DH. CH. *Q.E.D.*

Coroll. 1. Si FG × GH = GCq. vel FH × HG. HE :: R. T. erit EF tangens.

2. FG × GH = ¼ TR.

Prop.

a 23. 6.
b 34. 1 & 4. 6.
c 21. *hujus.*
d *cor. def. ad* 16. *hujus.*
e *hyp. cor.* 37.
f 15. 5.
g *prius.*
h *hyp. & sch.* 48. 2.
k 14. 6.

$:: AM : AL$. ff 19.5. ergo $GM \times AL =$
$= GA \times AM$. ergo $GM \times AL + GA \times AM$
$+ AM\,q = GM \times AL + GM \times AM$ ff
3.2 $= GM \times LM$ $(= 2\,GA \times AM + AM\,q)$
$= BM \times AM$. ff 3.2.

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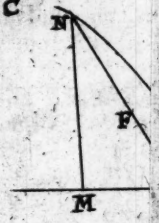

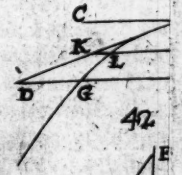

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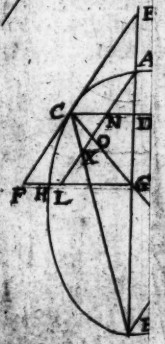

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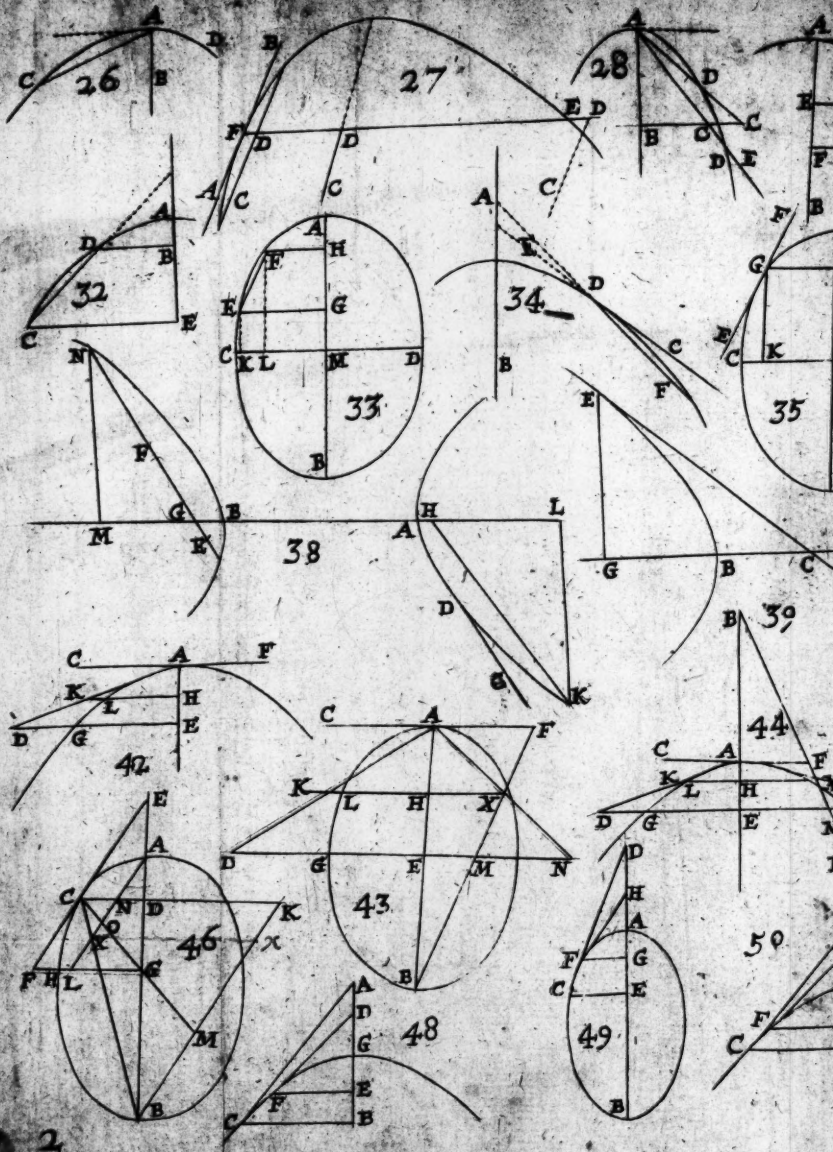
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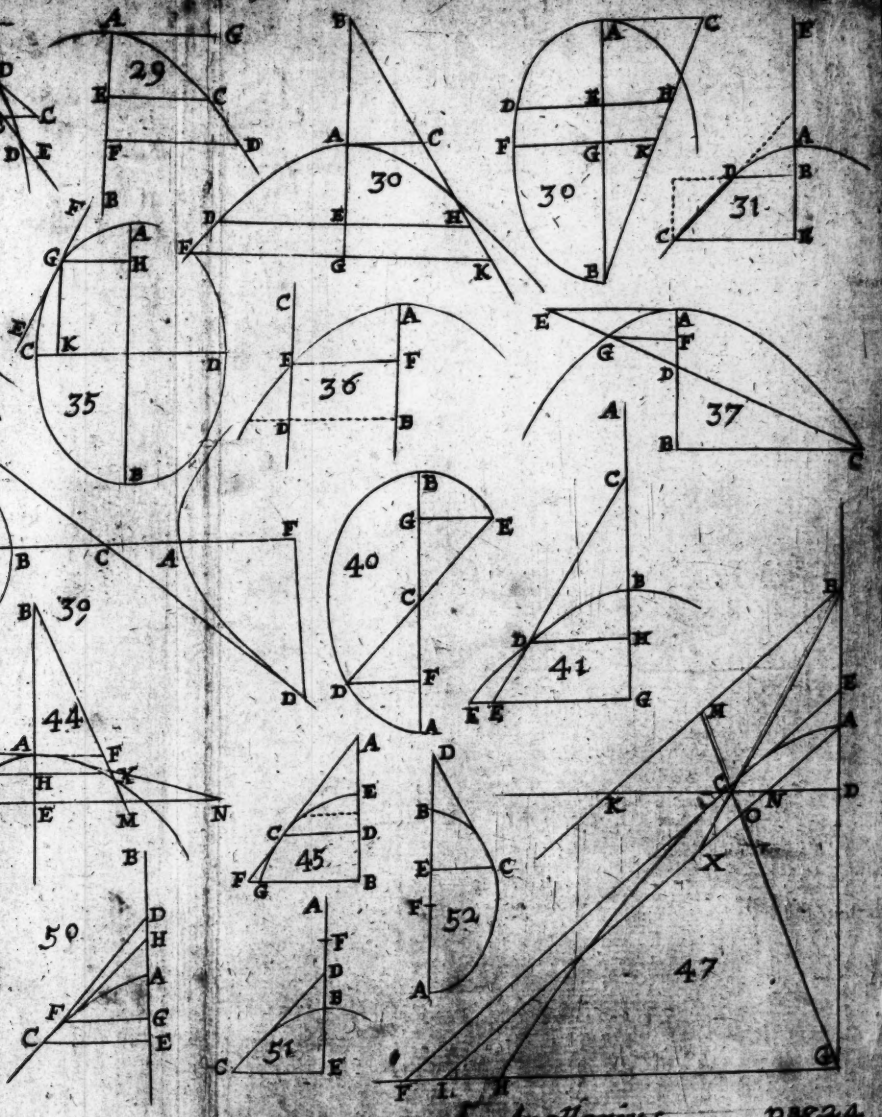

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Prop. XXXIX.

Si hperbolen, vel ellipſin, vel circuli circumferentiam contingens recta linea (C D) cum diametro (A B) conveniat (in D); & à tactu (C) ad diametrum (A B) ordinatim applicetur linea (C E); ſumptâ quâvis lineâ ex duabus, quarum altera (E F) interjicitur inter applicatam (C E), & ſectionis centrum (F); altera (E D) inter applicatam, & contingentem (C D); habebit ad eam applicata (C E) proportionem compoſitam ex proportione, quam habet altera dictarum linearum (E F, E D) ad applicatam (C E), & ex proportione, quam rectum figuræ latus habet ad tranſverſum. Fig. 57. 58.

Sit E F. C E :: G. E D. [a] vel E F × E D = C E × G. ergo C Eq. C E × G [b] (C E. G) [c] :: (C Eq. E F × E D :: [d]) R. T. atqui C E. E D [e] = C E. G + G. E D ([f] + E F. C E.) ergo C E. E D = R. T. + E F. C E. *Q. E. D.*

a 16. 6.
b 1. 6.
c 7. 5.
d 37. *hujus.*
e 5. *def.* 6.
f *conſtr.*

Prop. XL.

Si hyperbolen, vel ellipſin, vel circuli circumferentiam recta linea (A H) contingens (in A) conveniat cum ſecunda diametro (D E); & à tactu (A) ad eandem diametrum (D E) applicetur linea (A G) æquidiſtans alteri diametro (B C); ſumptâ quâlibet lineâ ex duabus; quarum una (G F) inter applicatam (A G) & ſectionis centrum (F) interjicitur, altera (G H) inter applicatam, & contingentem: habebit ad ipſam applicata (A G) proportionem compoſitam ex proportione, quam habet tranſverſum figuræ latus ad rectum, & ex ea quam altera dictarum linearum (G F, G H) habet ad applicatam (A G). Fig. 59. 60.

Sit G H. A G :: K. G F. [a] vel G H × G F = A G × K. ergo A Gq. G H × G F [b] (T. R.) [c] :: (A Gq. A G × K [d] ::) A G. K. atqui A G. G F [e] = A G. K + K. G F [f] (+ G H. A G). ergo A G. G F = T. R. + G H. A G. *Q. E. D.*

a 16. 6.
b 38. *hujus.*
c 7. 5.
d 1. 6.
e 5. *def.* 6.
f *conſtr.*

Prop. XLI.

Si in hyperbola, vel Ellipſi, vel circuli circumferentia ordinatim applicetur recta linea (C D) ad diametrum (A B); & ab applicata (C D), & ea (E A) quæ ex centro, deſcribantur parallelogramma æquian- Fig. 61. 62.

augula (D G, A F); habeat autem applicata (C D) ad reliquum latus (C G) parallelogrammi (D G) proportionem compoſitam ex proportione, quam habet ea quæ ex centro (E A) ad reliquum latus (E F), & ex proportione, quam rectum figuræ ſectionis latus (R) habet ad tranſverſum (T); parallelogrammum factum à lineâ (E D) quæ inter centrum (E) & applicatam (C D) interjicitur, ſimile parallelogrammo (A F), quod fit ab ea (E A) quæ ex centro, in hyperbola quidem majus eſt, quàm parallelogrammum (D G) ab applicata (D C), parallelogrammo (A F) ab ea (E A) quæ ex centro; In ellipſi verò, & circuli circumferentia, unà cum parallelogrammo (D G) quod fit ab applicata, æquale eſt parallelogrammo (A F) ab ea, quæ ex centro.

a 21. *hujus.*
b 1. 6.
c 9. 5.
d *conſtr.*
e *hyp.*
f 5. *def.* 6.
g 1. 6.
h *priùs & 7. 5.*
k 22. 6.
l 6. 2.
m 19. 5.
n 5. 2.
o 22. 6.

Fiat R. T [a] (hoc eſt D Cq. B D × D A) :: D C. C H [b] (hoc eſt D Cq. D C × C H). [c] ergo B D × D A = D C × C H. item D C. C H + A E. E F [d] = (R. T + A E. E F [e] = D C. C G [f] =) D C. C H + C H. C G. quare A E. E F [b] (hoc eſt A Eq. A E × E F) :: (C H. C G [g] :: C H × D C. C G × D C [h] ::) B D × D A. C G × D C. permutandóq; B D × D A. A Eq :: (C G × D C. A E × E F [k] ::) *p g r.* D G. F A. ergo in hyperbola componendo *p g r.* D G + F A. *p g r.* F A :: D Eq. [l] (B D × D A + A Eq). A Eq.

In Ellipſi verò & circulo, permutando A Eq *p g r.* F A :: (B D × D A. *p g r.* D G [m] ::) D Eq [n] (A Eq — B D × D A). *p g r.* F A — D G & permutando D Eq. A Eq :: *p g r.* F A — D G. *p g r.* F A. quare ſi fiat ex D E parallelogrammum ſimile ipſi A F [o]; liquet propoſitum.

Coroll. Quæ de parallelogrammis oſtenſa ſunt, eadem valent in trigonis horum dimidiis.

Prop. XLII.

Fig. 63.

Si parabolen contingens recta linea (C A) cum diametro (A B) conveniat in A; & à tactu (C) ad diametrum (A B) ordinatim applicetur linea (C H); ſumpto autem quovis puncto (D) in ſectione, applicentur ad diametrum duæ lineæ (D E, D F), altera quidem (D E) æquidiſtans contingenti (C A); altera vero (D F) æquidiſtans ei (C H), quæ à tactu (C) ordinatim eſt appplicata; triangulum (E D F) quod ab ipſis conſtituitur, æquale erit parallelogrammo (F G) contento linea (C H) à tactu applicata, & ea (F B), quæ interjicitur inter æquidiſtantem (D F) & ſectionis verticem (B.)

a 35. *hujus.*
b *ex.* 41. 1.

Nam ob A H [a] = 2 H B, [b] erit triang. C H A = *p g r.* H G. quare

p g r.

pgr. H G. triang. D F E [c] :: (triang. C H A. D F E [d] :: C Hq. D F q [e] :: H B. F B [f] ::) pgr. H G. pgr. F G. [g] ergo triang, D F E = pgr. F G. Q. E. D.

c 7. 5.
d 22. 6.
e 20. hujus.
f 1. 6. g 9. 5.

Prop. XLIII.

Si hperbolen, vel ellipsin, vel circuli circumferentiam contingens recta linea (E D) connveniat cum diametro (A B); & à tactu (E) ad diametrum (A B) ordinatim applicetur linea (E F); huic verò per sectionis verticem (B) ducatur æquidistans (B L), quæ cum linea (E C) per tactum (E) & centrum (C) conveniat in (L); & sumpto in sectione aliquo puncto (G), ab eo ad diametrum ducantur duæ lineæ (G H, G K) una quidem (G H) æquidistans contingenti (E D), altera vero (G K) æquidistans ei (E F), quæ à tactu applicata est: triangulum (G K H) ab ipsis factum in hyperbola minus erit, quàm triangulum (C K M) quod abscindit linea (C E) per centrum & tactum ducta, triangulo (C B L) ab ea (C B) quæ ex centro simili abscisso: in ellipsi verò, & circuli circumferentia una cum triangulo (C K M) abscisso ad centrum æquale erit triangulo (C B L), simili abscisso, quod describitur ab ea (C B) quæ ex centro.

Fig. 64. 65. 66.

Nam G K K H [a] = (E F. F D [b] = C F. F E + R. T [c] =) C B. B L + R. T. unde triang. G H K, æquatur excessui triangulorum C K M, B C L. Q. E. D.

a 4. 6.
b 39. hujus.
c 4. 6.
d cor. 41. hujus.

Coroll. Triang. G K H = 4 laterum K B L M.

Schol.

Triang. C B L = triang. C D E.

Vid. Eut. Vid: Not in pag: figurarum

Prop. XLIV.

Si unam oppositarum sectionum contingens recta linea (F G) contingens cum diametro (A B) conveniat (in G); à tactu verò (F) ad diametrum ordinatim applicetur linea (F O); atque huic per alterius sectionis verticem (B) ducatur æquidistans (B L), ut conveniat cum linea (F C) per tactum (F) & centrum (C) ductâ; sumpto autem quovis in sectione puncto (N), applicentur ad diametrum duæ lineæ (N K, N H), quarum altera (N K) æquidistet contingenti (F G), altera æquidistet ei (F O) quæ à tactu ordinatim applicata est; triangulum (N H K), ab ipsis factum, minus est, quàm triangulum (C M H) quod abscindit applicata ad centrum sectionis, triangulo simili (C B L), abscisso ab ea (C B) quæ ex centro.

Fig. 67.

Productâ F C, ut occurrat ſectioni B in E, per E ducatur tangens E D, & ordinatim applicetur E X. Eſtque O C × C G [a] = (A C q [b] = B C q [a] =) X C × C D. ergo cùm ſit O C [c] = X C, erit C G = C D. item F C [c] = E C & verticales anguli ad C [d] pares ſunt: [e] ergo ang. C G F = ang. C D E. [f] unde D E ad F G, & [g] proinde ad N K parallela eſt. [h] ergo triang. C M H — C B L = triang. N H K.

a 37. *hujus.*
b *hyp.*
c *cor.* 29. *hujus.*
d 15. 1.
e 4. 1.
f 27. 1.
g 30. 1.
h 43. *hujus.*

Coroll. C D = C G.

Coroll. Tangens E D tangenti F G æqualis & parallela eſt: & converſim; ſi E D tangenti F D æqualis vel parallela ſit, etiam E D tanget oppoſitam ſectionem.

Prop. XLV.

Fig. 68. 69. 70. 71.

Si hyperbolen, vel ellipſin, vel circuli circumferentiam contingens recta linea (C M L) cum ſecunda diametro (H D) conveniat (in L); & à tactu (C) ad eandem diametrum applicetur linea (C D), æquidiſtans alteri diametro (A H); & per tactum (C) & centrum (H) ducta linea (C H) producatur; ſumpto autem in ſectione quovis puncto (B), ad ſecundam diametrum (H D) ducantur duæ lineæ (B E, B F), quarum una (B E) contingenti (C L), altera (B F) applicatæ (C D) æquidiſtet; triangulum (B F E) quod ab ipſis conſtituitur, in hyperbola quidem majus eſt, quàm triangulum (G F H) abſciſſum ab applicata ad centrum, triangulo (L C H), cujus baſis eſt linea contingens (C L), & vertex ſectionis centrum (H): in ellipſi verò & circuli circumferentia, unà cum triangulo abſciſſo (G F H) æquale eſt triangulo (L C H), cujus baſis eſt linea contingens (C L) & vertex ſectionis centrum (H).

Ducantur C K, B N ad D H parallelæ. & trigonum ad A H, (ſimile trigono (C D L) appelletur P. eſtque C K. K H [a] = (M K. K C + R. T [b] =) C D. D L + R. T. quare in hyperb. triang. C D L (C D H + C L H) [c] = (triang. C K H + P [d] =) triang. C D H + P. [e] unde P = triang. C L H. Cæterum ob B N. F G [f] = (F H. F G. [g] = D H. D C [f] = C K. K H [h] = C D. D L + R. T [i] =) B F. (N H) F E + R. T. [e] Erit triang. B F E = triang. G H F + P, [m] ergo triang. B F E = triang. G H F + triang. C L H.

Simili diſcurſu, in ellipſi erit triang. B F E + G H F = triang. C L H.

a 39. *hujus.*
b 4. 6.
c *cor.* 41. *hujus.*
d 34. 1. & 1. *ax.*
e 3. *ax.*
f 7. 1.
g 4. 1.
h *priùs.*
i 4. 6.
m 2. *ax.* 1.

Prop.

Prop. XLVI.

Si parabolen contingens recta linea (C A) cum diametro (A B) conveniat (in A); quæ per tactum (C) duciur diametro æquidiſtans (H C M) ad eaſdem partes ſectioni lineas (L F) in ſectione ductas, quæ æquidiſtant contingenti (C A), bifariam ſecabit (in N.) Fig. 72.

Ordinatim applicentur B H, F G K, L M D. Eſtque triang. E L D [a] = *pgr.* B M. & triang. E F G [a] = *pgr.* B K. [b] ergo 4 lat. F L D G = *pgr.* G M. auferatur commune N M D G F; [b] manéntque trigona N M L, F K N æqualia. [c] eadémque ſimilia ſunt. ergo homologa latera N L, N F æquantur. *Q. E. D.*

[a] 42. *hujus.* [b] 3. *ax.* 1. [c] 29. 1. & 4. 6.

Coroll. 1.

In parabola omnes lineæ parallelæ diametro ſunt * etiam diametri: & viciſſim, omnes diametri ſunt parallelæ. * *def.* 10. *hujus.*

Coroll. 2.

Omnes contingenti æquidiſtantes ſunt ordinatim applicatæ ad diametrum per tactum ductam.

Prop. XLVII.

Si hyperbolen, vel ellipſin, vel circuli circumferentiam contingens recta linea (E D) cum diametro (A B) conveniat (in D); per tactum (E) & centrum (C) ducta linea (E C) ad eaſdem partes ſectioni, quæ in ſectione ducuntur contingenti (E D) æquidiſtantes (G N) bifariam ſecabit (in O.) Fig. 73. 74.

Ordinatim applicentur N F X, B L, G M K. Eſtque triang. H N F [a] = 4 lat. L B F X. & triang. G H K [a] = quadrilat. L B K M. [b] ergo 4 lat. N G K F = M K F X. commune auferatur O N F K M, [b] manent trigona O M G, O X N æqualia. [c] eadem vero ſimilia ſunt. ergo N O = G O. *Q. E. D.*

[a] *cor.* 43. *hujus.* [b] 3. *ax.* 1. [c] 29. 1. & 4. 6.

Coroll. C E eſt diameter ſectionis cujuſlibet ex his.

Prop. XLVIII.

Si unam oppoſitarum ſectionum contingens recta linea (L K) cum diametro (A B) conveniat (in K); & per tactum (L), & centrum (C) linea (L C) producta alteram ſectionem ſecet (in E); quæ in altera ſectione Fig. 75.

sectione ducta fuerit contingenti (L K) æquidistans (G N) à lineâ (L C) productâ bisecabitur (in O.)

Ducatur tangens E D ; [a] estque E D ad L K parallela ; [b] ac ideò ad N G. [c] ergo O N = O G. *Q. E. D.*

a *cor.* 44. *hujus.*
b 30. 1.
c 47. *hujus.*

Lemma.

Fig. 76. 77. Sit triangulum N L K æquale parallelogrammo L C, & ang. K L N = D L P : erit K L × L N = 2 D C × L D.

Compleatur enim *pgr.* L R. & productâ L P, fiat P T = L P. & compleatur *p g r.* D T. erit que *pgr.* L R [a] = *pgr.* D T. [b] unde K L. L D :: L T (2 D C). L N. [c] quare K L × L N = 2 D C × L D.

a 6. *ax.* 1.
b 14. 6.
c 16. 6.

Item, si D P fuerit trapezium trigono K L N æquale, erit K L × L N = L D × : C D + L P. Nam fiat P T = D C ; & compleatur *pgr.* D T. erit que *pgr.* D T = (2 D P = 2 triang. K L N =) *pgr.* L R. unde K L. L T. (L P + D C) :: L D. L N. quare K L × L N = L D × : D C + L P. *Q. E. D.*

Schol. Hinc K Nq. K Lq :: R. G (hoc est ut parameter axis ad parametrum diametri D N).

Fig. 78. Nam K Nq. K Lq [a] :: X Dq. D C q : & est X Dq [b] = R × B X. [c] & D C q = G × B X. [d] ergo K Nq. K Lq :: (R × B X. G × B X [e] ::) R. G.

a 4. & 22. 6.
b 11. *hujus.*
c p[illegible]
d 7. 5.
e 1. 6.
Eſt. 2. 7*mi* Apollonii.

Schol.

Not. Si R sit parameter axis B X ; erit G = R + 4 B X.

Nam sit B K ad D C parallela. ergo C B = D L = B X. | B T = 2 D L.

$$G \times \begin{cases} DL = BLq = \\ BX. \end{cases} \left| \begin{array}{l} DXq + BTq = R \times BX + 4BXq. \\ LTq. \end{array} \right.$$

ergo G = R + 4 B X.

Prop. XLIX.

Fig. 79. Si parabolen contingens recta linea (D C) cum diametro (B C) conveniat in (C) ; & per tactum (D) ducatur linea (F N) æquidistans diametro (C B) ; à vertice verò (B) ducatur æquidistans (B F) ei (D X), quæ ordinatim applicata est ; & fiat ut contingentis portio (E D) inter applicatam (B F) & tactum (D) interjecta ad æquidistantis portionem (D F), quæ itidem inter tactum (D) & applicatam (B F) interjicitur ; ita quædam recta linea (G) ad duplam contingentis (D C ;)

quæ

quæ (KL) à ſectione ducta fuerit contingenti (DC) æquidiſtans, ad lineam (FN), quæ per tactum ducitur diametro æquidiſtans, poterit rectangulum contentum inventâ lineâ (G), & eâ (LD), quæ inter ipſam (KL), & tactum (D) interjicitur. DE. | DF :: G. | BX. | 2 DC

Ordinatim applicentur DX & KNM. Eſtque CB [a] = (BX [b] = FD); [c] unde triang EBC = EFD, additóque communi DEBMN, erit DCMN [d] = (*pgr.* FMC [e] =) triang. KPM. ablatóque communi LPMN, erit *pgr.* LC [f] = triang. NLK. [g] unde KL × LN = 2DC × LD.

Itaq; G × LD. KL × LN [h] :: (G × LD. 2DC × LD [k] :: G. 2DC [l] :: ED. DF [m] :: KL. LN [n] :) KLq. KL × LN. [o] ergo G × LD = KLq.

Cor. Hinc DL eſt diameter, & G rectum latus ſectionis, cujus vertex D. [illegible] = $\frac{2DE \cdot DC}{BX}$, eſt rectum latus ſectionis, cujus vertex B.

a 35. *hujus.*
b 34. 1.
c *ex.* 20. 6.
d 2. *ax.*
e 42. *hujus.*
f 3. *ax.*
g *lemma præc.*
h 7. 5.
k 1. 6.
l *hyp.*
m 4. 6.
n 1. 6.
o 9. 5.

Prop. L.

Si hyperbolen, vel ellipſin, vel circuli circumferentiam contingens recta linea (ED) cum diametro (AB) conveniat, pérque tactum (E) & centrum (C) linea (EC) producatur; à vertice autem (B) ordinatim applicata (BG) conveniat cum ea (EC), quæ ducitur per tactum, & centrum; fiatque ut contingentis portio (EF) inter tactum (E) & applicatam BG ad portionem (EG) lineæ (EC) ductæ per tactum & centrum; quæ itidem inter tactum (E) & applicatam (BG) interjicitur, ita quædam recta linea (EH) ad duplam contingentis (ED); quæ (LM) à ſectione ducitur contingenti (ED) æquidiſtans ad lineam (EC) per tactum, & centrum ductam, poterit ſpatium rectangulum, quod adjacet inventæ lineæ (EH), latitudinem habens interjectam (EM) inter ipſam (LM) & tactum (E); in hyperbola quidem excedens figurâ ſimili contentæ lineâ (EK) dupla ejus (CE), quæ eſt inter centrum & tactum, & inventâ lineâ (EH), in ellipſi verò, & circulo deficiens eâdem. Fig. 80. 81. EF. EG :: EH. 2ED.

Ducatur LRN ad BG, & CSO ad KP parallelæ. Et ob EK [a] = 2EC, [b] erit EH = 2ES. ergo 2ES. 2ED [c] :: (EH. 2ED [d] :: FE. EG [b] :) LM. MR. porro ob triang. RNC [e] = CDE [f] (CGB) + LNX (in hyperbola), [g] vel triang. RNC + LNX = CDE (in ellipſi & circulo) erit trapezium MEDX [h] = triang. LMR.

a *hyp.*
b 4. 6.
c 7. 5.
d *hyp.*
e 43. *hujus.*
f *ſch.* 43. *hujus.*
g 3. *ax.* 2.

h *lem. ante* 49. LMR. h ergo LM×MR = EM×: ED + MX. Denique quia
k 4. 6. MO. ES k :: (MC. CE k ::) MX. ED. componendóque MO +
l 1. 6. ES. ES :: MX + ED. ED, erit permutando MO + ES. MX +
m *prius. et* 7. 1. ED (l hoc eſt EM×: MO + ES. EM×: MX + ED m vel EM
n 15. 5. ×: MO + ES. LM×MR) :: ES. ED n :: 2 ES. 2 ED o :: LM.
o *prius.* MR. l :: LMq. LM×MR. p ergo EM×: MO + ES = LMq.
p 9. 5. atqui ES q = (SH r =) OP. ergo EM×MP = LMq. *Q. E. D.*
q *prius.*
r 34. 1.

Cor. EK eſt diameter, & EH latus rectum ſectionis, cujus vertex E.

Prop. LI.

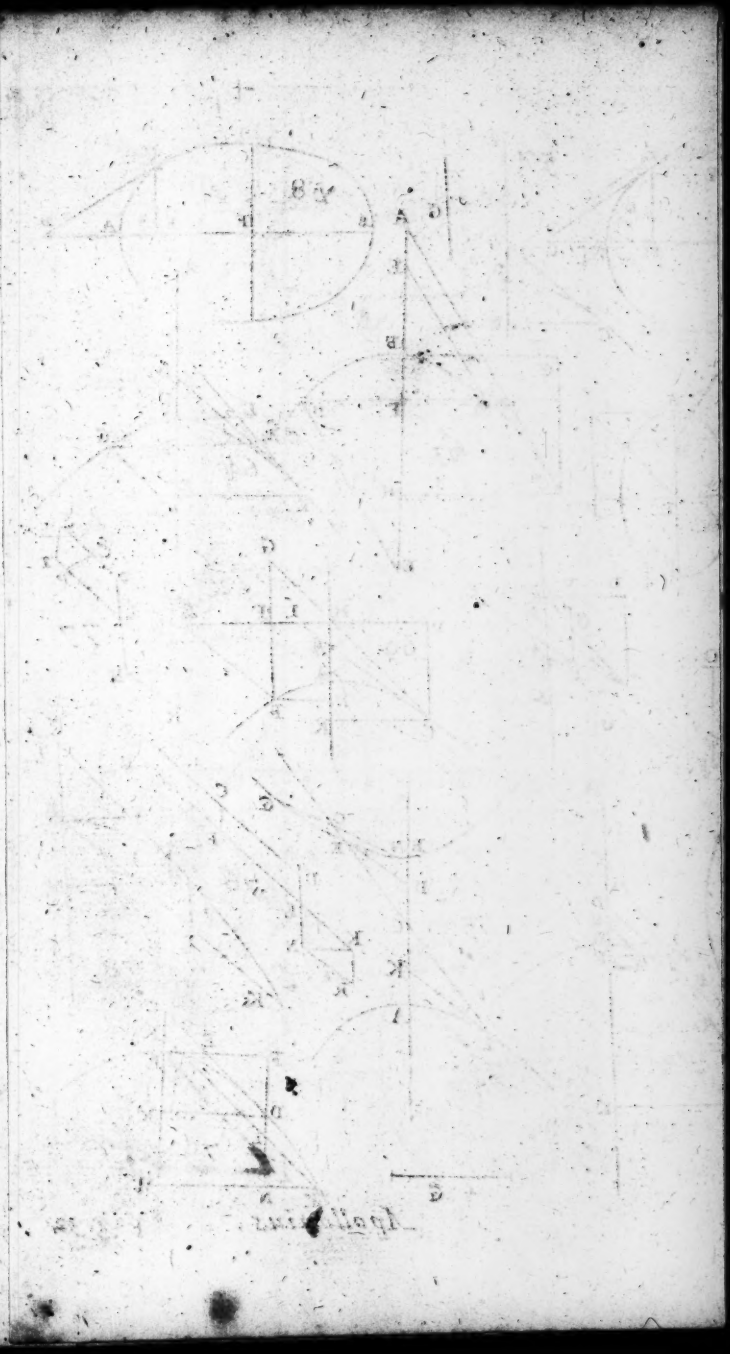
Fig. 82. Si quamlibet oppoſitarum ſectionum contingens recta linea (CD) cum diametro (AB) conveniat; perque tactum (C) & centrum (E) linea (CE) producatur uſque ad alteram ſectionem; à vertice verò (B) ducatur linea (BG) æquidiſtans ei, quæ ordinatim applicata eſt, conveniénſque cum linea (CE) per tactum & centrum ducta; & fiat ut contingentis portio (LC) inter applicatam & tactum ad portionem (CG) lineæ ductæ per tactum & centrum, quæ inter tactum & applicatam interjicitur, ita quædam recta linea (K) ad duplam contingentis (CD), quæ in altera ſectione ducitur, æquidiſtans contingenti (FM) ad lineam (FE) per tactum, & centrum ductam, poterit rectangulum, quod adjacet inventæ lineæ (K) latitudinem habens lineam, quæ eſt inter ipſam, & tactum (F), excedénſq; figurâ ſimili ei, quæ lineâ (CF) inter oppoſitas ſectiones interjectâ, & inventâ (K) continetur.

a *cor. 44. hujus.* Ordinatim applicetur AXN. a Súntque FM, CD æquales & par-
b *conſtr.* allelæ: b itémque AN, BG parallelæ ſunt. ergo FX. FN c :: (LC.
c 4. 6. CG :: d K. 2CD c ::) K. 2FM; unde quæcunque à ſectione AF ad
d *hypoth.* productam EF contingenti FM ducuntur parallelæ, f poterunt re-
e 7. 5. ctangulum contentum ipsâ K, & interjecta inter iſtas; & punctum
f 50. *hujus.* F, excedéntque figurâ ſimili ei, quæ rectâ CF, & ipsâ K continetur.

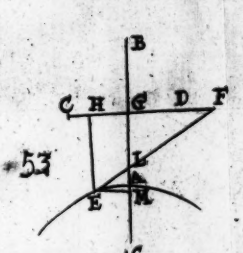
Coroll.

a 46. *hujus.* Itaque his demonſtratis, liquet in a parabola unamquamque rec-
b 47. tarum linearum, quæ diametro ex generatione ducuntur æquidiſtantes,
c 48. diametrum eſſe; In b hyperbola verò & ellipſi & c oppoſitis ſectionibus, unamquamque earum, quæ per centrum ducuntur.

d 49. Et in d parabola quidem applicatas ad unamquamque diametrum,
e 50. æquidiſtantes contingentibus poſſe rectangula ipſi adjacentia: in e Hyperbola,



Vel. juxta Schol. Prop: 43
Triang: CBL = triang: CDE
quia EF, FD æquatur exces-
-sui triangulorum EFC,
CBL. per demonstrationem
ibi traditam mutatis mutandis.
atqui EF, FD æquatur exces-
-sui triangulorum EFC, CDE.
ut patet ergo CBL = CDE.



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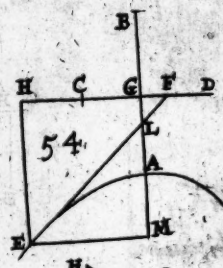

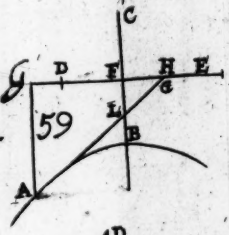

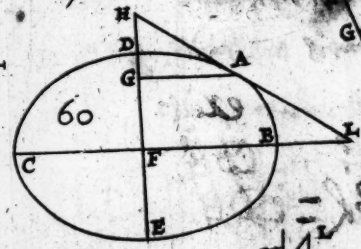

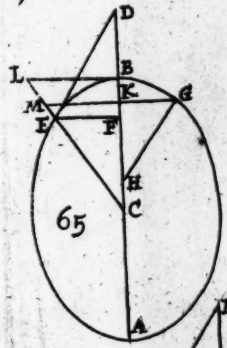

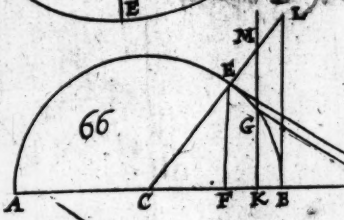

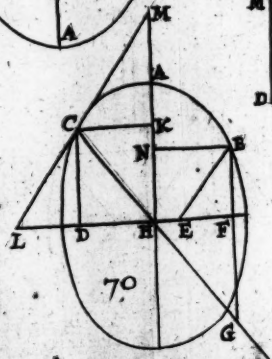

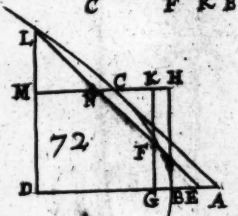

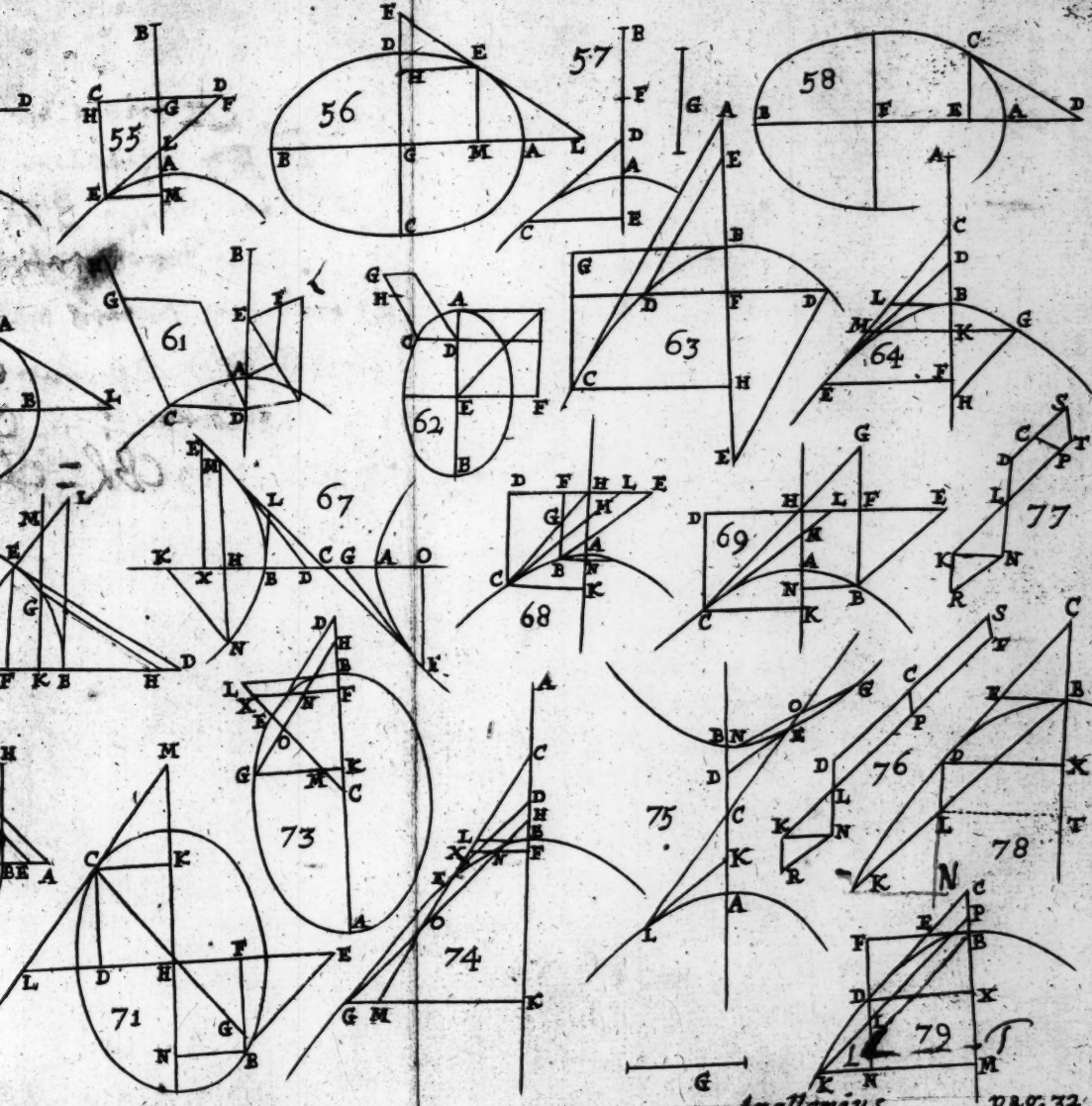
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Apollonius. pag. 32

perbola, & [f] oppositis rectangula adjacentia ipsi, quæ excedunt eâdem figurâ, in ellipsi autem, quæ eâdem deficiunt. Postremò quæcunque circa sectiones, adhibitis principalibus diametris demonstrata sunt, & aliis diametris assumptis eadem contingere. [f] 51. *hujus.*

Prop. LII. Probl. 1.

Datâ in plano rectâ lineâ (A B) ad unum punctum (A) terminatâ, invenire in plano coni sectionem, quæ Parabole appellatur, ita ut ejus diameter sit data linea (A B), vertex lineæ terminus (A); quæ verò à sectione ad diametrum (A B) in dato angulo applicatur, possit rectangulum contentum lineâ, quæ est inter ipsam & sectionis verticem (A), & alterâ quâdam datâ lineâ (Z). Fig. 83.

Datus angulus primò rectus sit. Producatur A B ad E, ita ut A E ⊏ ¼ Z. Sitque Z. Y [a] :: Y. A E. unde Z. A E [b] :: Yq. A Eq. ergo cum Z [c] ⊐ 4 A E, [d] erit Yq ⊐ 4 A Eq, & proinde Y * ⊐ 2 A E. [e] ergo ex Y, & duabus AE constitui poterit triangulum. Fiat ergo EAF, rectum subjecto plano, ita ut AF = AE, & E F = Y; ducantúrque A K ad E F, & F K ad E A parallelæ ([f] unde A K [f] = E F = [g] = Y, & F K [f] = E A [g] = F A). Tum concipiatur conus, cui vertex F, basis circulus super diametrum A K, rectus plano A F K; erit is conus rectus (ob F K = F A). Secetur conus plano ad circulum A K parallelo, [h] facientíque proinde sectionem M X N circulum, plano M F N (vel F A K) rectum; horúmque communis sectio sit recta M N, [h] diameter nempe circuli M X N; communis autem sectio subjecti plani, & circuli sit recta X L. Quum igitur tam circulus M X N, quàm subjectum planum recta sint triangulo M F N, [k] erit istorum communis sectio X L recta trigono M F N, [l] ideóque rectis (quæ in eo) M N, A B perpendicularis. Ex quibus constat planum per A B, X L ductum facere in cono sectionem, quæ parabole dicitur (juxta conditiones in 11ª hujus præscriptas) cujus diameter A B, lineæque ad hanc à sectione ordinatim ductæ ad rectos angulos applicentur, utpote ad X L parallelæ. Porro ob Z, Y; A E [m] (hoc est Z, A K, A F) [n] ÷÷, [o] erit Z. A F :: A Kq. A F q (A F * F K). [p] unde Z est rectum latus. Ergò factum. 1. *Cas.*

[a] 13. 6. [b] *cor.* 20. 6. [c] *constr.* [d] 14. 5. [e] 4. 2. [f] 22. 1. [g] 34. 1. [h] *constr.* [k] 4. *hujus.* [l] 19. 11. [m] 3. *def.* 11. [n] *prius.* [o] *constr.* [p] *cor.* 20. 6. 11. *hujus.*

Sed datus angulus non sit rectus, sitque ei æqualis H A E; & fiat A H = ½ Z; & per H ducatur H E ad A E perpendicularis, & per E ad H B parallela E L, & per A ad E L perpendicularis A L: tum bisectâ E L in K, per K ducatur ipsi E L perpendicularis M K F G; 2. *Cas.* Fig. 84.

sitque A Lq [a] = L K * K M. Datis igitur rectâ K L positione, & rectâ K M magnitudine, & recto angulo, describatur (ut modò ostensum) parabole, cujus diameter K L, vertex K, & rectum latus K M. Transibit hæc per A ([b] ob A Lq = K L * K M) & A E [c] continget ipsam, (ob L K = K E) & H A est diameter [d] (quia ad E L parallela), [e] & quæ ad A E parallelæ, bisecantur ab A B, [f] inque angulo H A E applicantur: & ob trigona A G F, A E H [g] similia (quia anguli A E H, A G F recti, & H A E communis), est F A. A G [h] :: (H A. A E [i] ::) 2 H A. 2 A E, [k] hoc est F A. A G :: Z. 2 A E. [l] unde Z rectum erit latus sectionis. Quæ *E. F.*

a 11. 6.
b 11. *hujus.*
c 33. *hujus.*
d *cor.* 46. *huj.*
e 45. *hujus.*
f *constr.*
g 4. 6.
h 15. 5.
k *constr.*
l 49. *hujus.*

Prop. LIII. Probl. 2.

Fig. 85.

Datis duabus rectis lineis (A B, B C) terminatis, quæ ad rectos inter se angulos constituantur; & alterâ (A B) productâ ad * easdem partes angulo recto, invenire coni sectionem, quæ hyperbole dicitur; in eodem plano, in quo sunt datæ lineæ (A B, B C), ita ut producta (A B) sit diameter sectionis, & vertex punctum (B), quod ad angulum (A B C) consistit; quæ verò à sectione ad diametrum ordinatim applicatur, angulum faciens, æqualem dato, possit rectangulum, quod ad jacet alteri lineæ (B C), latitudinem habens lineam interjectam inter applicatam, & sectionis verticem (B), excedénsque figurâ simili, & similiter positâ ei, quæ datis à principio lineis (A B, B C) continetur.

* *hoc est versùs* D.

1. *Cas.*

Sit datus angulus primò rectus; & super lineam A B planum attollatur, rectum subjecto plano, in quo circa A B describatur circulus, * ita ut ductâ diametro E K L ad A B perpendiculari, non sit ratio E K ad K L major eâ, quam habet A B ad B C. Fiat igitur E K. K M [a] :: A B. B C. & per M ducatur M F ad A B parallela; junctísque A F, E F, B F, per B ducatur B X ad E F parallela. Itaque ob ang. A X B [b] = (A F E [c] = E F B [b] =) F B X, [d] erit F B = F X. Concipiatur jam conus, cujus vertex F, basis circulus super diametrum B X, rectus trigono F B X. Erit is conus rectus (ob F B = F X). Producantur F B, F X, M F, & secetur conus plano, ad circulum B X parallelo, [e] facienti proinde circulum G P H R rectum plano F X B, cujúsq; diameter G H communis sit istorum planorum sectio. Sit verò P D R communis sectio circuli G R H, & subjecti plani. Et quoniam tam circulus G R H, quàm subjectum planum recta sunt trigono F G H, [f] erit horum communis sectio P D R eidem trigono F G H recta; [g] idcóque rectis, quæ in eo, G H, D B perpendicularis. Ex quibus liquet

* *vid. not.* 1.
a 12. 6.
b 29. 1.
c 27. 3.
d 6. 1.
e 4. *hujus.*
f 19. 11.
g 3. *def.* 11.

liquet coni ſectionem P B R (juxta conditiones in 12ma hujus præſtitutas) eſſe hyperbolen, cujus vertex B, & ordinatæ ad diametrum A B in angulo recto applicentur, quippe omnes ad ipſam P R parallelæ. Porro ob A B. B C [h] :: (E K. K M [k] :: N E. N F [i] :: N E × N F [m] (N A × N B). N Fq [n] = N A. N F [o] (O F. F G) + N B. N F ([o] O F. F H) [n] =) O Fq. O G × O H [p] :: A B. B C. [q] erit A B tranſverſum latus, & B C rectum.

h *conſtr.*
k 2. 6.
l 1. 6.
m 35. 3.
n 23. 6.
o 4. 6.
p 11. 5.
q 12. *hujus.*

Sin datus angulus non ſit rectus. Datæ ſint rectæ A B, A C, & angulus B A H per dato. Biſecetur A B in D, & deſcripto ſuper A D ſemicirculo, occurrat G F ad A H parallela, * faciens G Fq. D G × G A :: A C. 2 A D; junctaq; F D, [a] fiat F D. D L :: D L. D H; & ſumptâ D K = D L, [b] fiat L F. A F :: A F. F M; & connectatur K M, & per L ducatur N L X ad K F perpendicularis. Deſcribatur tunc Hyperbole (juxta modò oſtenſa), cujus vertex L, tranſverſus axis K L, rectum latus L N; [c] tranſibit hæc per A [d] (ob L F × F M = A Fq.) & A H ſectionem [e] continget (ob F D × D H = D Lq) [f] & proinde A B eſt diameter ſectionis. Porro, quum ſit A C. 2 A H + F G. G D [g] = (A C. 2 A H. + A H. A D [h] = A C. 2 A H + 2 A H. 2 A D [k] = A C. 2 A D [l] = F Gq. G A × G D [m] =) F G. G A + F G. G D. Erit A C. 2 A H :: (F G. G A [n] ::) O A. A X. [o] unde A C eſt rectum latus. Ergo factum.

vid. not. 2.
a 13. 6.
b 11. 6.
c 12 *hujus.*
d *conſtr.* & 17.
e 37. *hujus.* (6.
f *cor.* 47. *hujus.*
g 4. 6.
h 15. 5.
k 5. *def.* 6.
l *conſtr.*
m 23. 6.
n 4. 6.
o 50. *hujus.*

Notæ.

1. Deſcribitur circulus circa A B, ita ut E K. K L :: A B. B C, hoc pacto. Fig. 87.

Fiat utcunque Z K. K Y [a] :: A B. B C. & biſectâ Z Y in V, centro V, per Z, & Y deſcribatur circulus, ſecans ipſam A B (ſi opus eſt, productam) in S, & T; connexiſque S Y, S Z, per A ducantur ad has parallelæ A L, A E. ergo quum angulus Y S Z rectus ſit, [b] erit quoque angulus L A E rectus. [c] ergo ſuper diametrum L E deſcriptus circulus tranſibit per A, [d] ideóque per B, [e] quia K B = K A. eſtq; E K. Z K [f] :: (A K. S K [g] ::) L K. Y K. & permutando E K. L K :: (Z K. Y K [g] ::) A B. B C.

a 12. 6.
b 29. 1. *et* 2. *ax.*
c *conv.* 31. 3.
d *cor.* 1. 3.
e *hyp.*
f 4. 6.
g *conſtr.*

2. Quomodo autem ducatur G F ad A H parallela, faciens G Fq ad D G × G A in data ratione (puta R ad S), ita conſtabit. Sumpto Z centro circuli, ducatur Z Y ad A H T perpendicularis, & ab occurſu Y, ducatur Y Q ad A H parallela, [a] quare Y Q tangit circulum. Fiat verò Q V. V Y [b] :: S. R − S. & productâ Q Y, ſumatur Y K = Y V; con- Fig. 88.

a *cor.* 16. 3.
b 12. 6.

connectantúrque Z K, Z V circulum ſecantes in P, & F; conjunctaq; P F protrahatur ad G. dico factum.

c 4. 1. d 2, 6. e 30. 1. f *conſtr.*

Nam ob V Y = K Y, & angulos ad Y rectos, [c] erit Z V = Z K. item Z F = Z P, [d] ergo F P ad V K, [e] hoc eſt ad A H, eſt parallela. Et quoniam S. $\frac{R-S}{2}$ [f] :: Q V. V Y, erit duplando conſequentes, S. R—S :: Q V. V K. & invertendo R—S. S :: V K. Q V. & componendo R. S :: (Q K. Q V *:: G P. G F [g] :: G P × G F. G Fq [h] ::) D G × G A.

g 1. 6. h 36. 3. & 7. 5

G Fq. ac inverſè S. R :: G F q. D G × G A. *Q. E. F.*

Prop. LIV. Probl. 3.

Fig. 89.

Datis duabus rectis terminatis (A B, A C), atque ad rectos inter ſe angulos, invenire circa diametrum ipſarum alteram (A B) coni ſectionem, quæ Ellipſis appellatur, in eodem plano, in quo ſunt datæ lineæ (A B, A C), ita ut vertex ſit punctum (A) ad rectum angulum (B A C); & à ſectione ad diametrum (A B) applicatæ in angulo dato poſſint rectangula adjacentia alteri lineæ (A C), quæ latitudinem habeant lineam inter ipſas, & verticem (A) ſectionis interjectam, deficiántque figurâ ſimili, & ſimiliter politâ ei, quæ datis rectis lineis (A B, A C) continetur.

1. *Caſ.*

Sit datus angulus primò rectus, & ex A B planum attollatur, rectum ſubjecto plano, in quo circa A B deſcriptum ſit circuli ſegmentum A D B; & biſecetur arcus A D B in D, unde connectantur D A, D B; & fiat A X = A C; & per X ducatur X O ad B D parallela, & per O ipſa O F ad A B parallela, & juncta D F occurrat protractæ B A in E. Jungantur F A, F B, & producantur, perque punctum G (utcunque ſumptum in F A) productâ) ducatur ipſi E D parallela G H, productæ A B occurrens in K, productæque F O in L. Eſtque ideo ang. H G F [a] = (ang. E F A [b] = F A D [c] (F B D) + F D A [d] (F B A) = F B D + F B A [c] = A B D [c] = D F B [a] =) G H F [f] = H G F. [g] unde F G = F H. Itaque ſuper G H deſcribatur circulus G H N rectus trigono H F G, ſitque baſis coni recti, habentis verticem F. Et quia tam circulus G H N, quàm ſubjectum planum recta ſunt plano H F G, [h] erit ipſorum communis ſectio (K M) plano eidem, [k] rectíſque idcirco G K, A K perpendicularis.

a 29. 1. b 32. 1. c 26. 3. d 21. 3. e 19. *ax.* 1. f 1. *ax.* g 6. 1. h 19. 11. k 3. *def.* 11.

Liquet igitur planum per A K M [l] facere in cono ellipſin, cui diameter A B, ad quam ordinatæ omnes perpendiculariter applicentur, quippe ipſi K M parallelæ: porro, ob F Lq. G L × L H [m] = (F L. G L

l 13. *hujus.* m 13. 6.

GL[n] (AK. KG, [n] vel AE. EF) + FL. LH ([n] BK. KH, [n] vel BE. EF) = AE. EF + BE. EF[m] = AE × BE. EFq [o] = DE × EF. EFq [p] = DE. EFq = DA. AO :: qBA, AX[r] ::) BA. AC[s] :: FLq. GL × LH. erit AC rectum latus.

n 4. 6.
o 36. 3. & 7. 5.
p 1. 6. q 4. 6.
r *conſtr. et* 7. 5.
s 11. 5.

Sin diameter AB minor ponatur dato recto latere AC; biſectâ AB in D, ducatur FE biſecta quoque in D, ita ut ſit AC. FE [a] :: FE. AB. & ductâ FG ad AB parallela, ſit FE. FG [b] :: AC. AB ([c] hoc eſt) :: FEq. AEq :: FDq. [d] (FD × DE). ADq [e] :: FE. FG. Ducatur itaque (ut modo oſtenſum) ellipſis, cujus axis EF (& rectum latus FG; tranſibit hæc per A. (ob FD × DE. ADq [f] :: FE. FG) [g] ideóque per B ([d] ob AD = DB). item propter AC. AB :: FDq. AD × DB (ADq). [f] erit AC rectum latus.

2. *Caſ.* Fig. 90.
a 13. 6.
b 12. 6.
c *cor.* 20. 6.
d *conſtr.*
e 11. 5.
f 21. *hujus.*
g 30. *hujus.*

Sed datus angulus non ſit rectus; ſitque ei æqualis angulus BAD; biſectâque AB in E, circa AE deſcribatur ſemicirculus, in quo ad AD *ducatur parallela FG, faciens FGq. AG × GE :: CA. AB; & junctæ AF, EF producantur; [a] & ſit DE. EH :: EH. EF & ſumptâ EK = EH, factóque HF. FA [b] :: FA. FL, conjungatur KL, occurrens ductæ NM (per H ad AL) parallelæ. Tum (ex modò præoſtenſis) deſcribatur ellipſis, cujus axis tranſverſus ſit KH, & rectum latus HM. [c] tranſibit hæc utique per A [d] (ob HF × FL = FAq); & [e] idcirco per B (ob AE [c] = EB) ac ipſam [f] continget DA [g] (ob DE × EF = EHq). Item propter CA. 2DA. + FG. GE [h] = (CA. 2DA + DA. AE [k] = CA. 2DA + 2DA. AB [l] = CA. AB [m] = FGq. AG × GE [n] =) FG. AG + FG. GE. erit CA. 2DA *:: (FG. AG [p] ::) XA. AN. [q] ergo AC eſt rectum latus.

3. *Caſ.* Fig. 91.
* *vid. Not.*
a 13, 6. b 11. 6.
c 13. *hujus.*
d *conſtr.*
e 30. *hujus.*
f *conv.* 38 *huj.*
g *conſtr.* & 17. 6.
h 4. 6.
k 15. 5.
l 5. *def.* 6.
m *conſtr.*
n 23. 6.
*
p 4. 6.
q 50. *hujus.*

Notæ.

Quomodo verò duci poterit GF ad AD parallela, ita ut ſit GFq ad AG × GE in data ratione S ad R, ita conſtabit. Fig. 92.

Sumpto Z centro circuli, ducatur ad AD perpendicularis ZY, circulo occurrens in Y. & per Y ducatur QY ad AD parallela; [a] & proinde tangens circulum in Y, occurrenſque productæ ZA in Q.

a *cor.* 16. 3.

[b] Fiat autem $\frac{R-S}{2}$ S :: YQ. QV. productâque VY ſumatur YK = YV; & junctæ VZ, KZ producantur, adeò ut completo circulo occurrant punctis F, P, & connectatur FP, ſecans AE in G. Dico factum.

b 12. 6.

Nam

Nam ob ZV = ZK, & ZP = ZF, erit PF ad VK, hoc eſt ad AD parallela. item (ob $\frac{R-S}{2}$ S :: YQ. QV,) erit componendo $\frac{R+S}{2}$. S. :: YV, QV. & duplando antecedentes, R+S. S :: KV. QV. & dividendo R. S :: (KQ. QV :: PG. GF :: PG × GF. GFq ::) AG × GE. GFq. & inverse S. R :: GFq. AG × GE. *Q. E. F.*

c *conſtr. & 4. 1.* d *2. 6.* e *30. 1.* f *conſt.* g * h *1. 6.* k *35. 3. & 7. 5.*

Prop. LV. Probl. 4.

Fig. 93.

Datis duabus rectis terminatis (EB, BH), atque ad rectos inter ſe angulos, invenire oppoſitas ſectiones, quarum diameter ſit una (EB) datarum linearum; & vertices lineæ termini (E, B); applicatæ vero ab utraque ſectione poſſint ſpatia adjacentia alteri lineæ (BH), excedentiáque figurâ ſimili ei, quæ datis lineis (EB, BH) continentur.

a *53. hujus.*

[a] Deſcribatur hyperbole ABC, cujus diameter tranſverſa ſit BE, & rectum latus BH; & ordinatæ ad BE in dato angulo (G) applicentur. Ducatur quoque EK ad rectos ipſi BE, æqualiſque ipſi BH, & [a] deſcribatur itidem alia hyperbole DEF, cujus diameter ſit EB, rectum latus EK; ductæque ordinatim applicentur in angulo, qui deinceps ipſi G. Liquet igitur deſcriptas hyperbolas ABC, DEF fore ſectiones oppoſitas, habentes diametrum communem BE, & recta latera BH, EK inter ſe æqualia. *Q. E. F.*

Prop. LVI. Probl. 5.

Fig. 94

Datis duabus rectis lineis (AC, DE) ſeſe bifariam ſecantibus (in B), circa utramque ipſarum oppoſitas ſectiones deſcribere, ita ut rectæ lineæ (AC, DE) ſint conjugatæ diametri; & quarumlibet oppoſitarum ſectionum diameter poſſit figuram aliarum oppoſitarum.

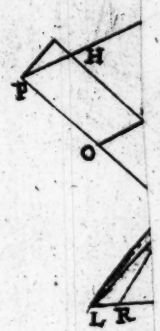
a *11. 6 et 17. 6.* b *55. hujus.* c *def. ad 16. hujus.* d *conſtr.*

Sit AC × CL [a] = DEq; (vel AC, DE, CL ÷÷) [b] deſcribantúrque ſectiones oppoſitæ FAG, HCK, quarum tranſverſa diameter ſit AC, rectum latus CL; & ordinatim ductæ ad CA in dato angulo applicentur. [c] Erit harum ſecunda diameter ipſa DE; quâ AC. DE [d] :: DE. CL; [d] & DE ordinatim applicatis parallela biſecatur in B. [a] Sit pariter DE × DR = ACq. & [b] deſcribantur ſectiones oppoſitæ MDN, OEX, quarum tranſverſa diameter DE, & DR rectum latus; ductæque ordinatim ad DE in dato angulo applicentur; [c] eritque harum ſecunda diameter AC, ob DE. AC [d] :: AC. DR, & AC biſectam in B. Ergo factum.

Deſinitio.

Vocentur autem hujuſmodi ſectiones *Conjugatæ*.

APOL-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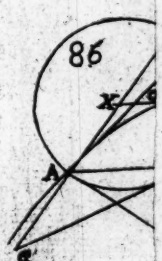
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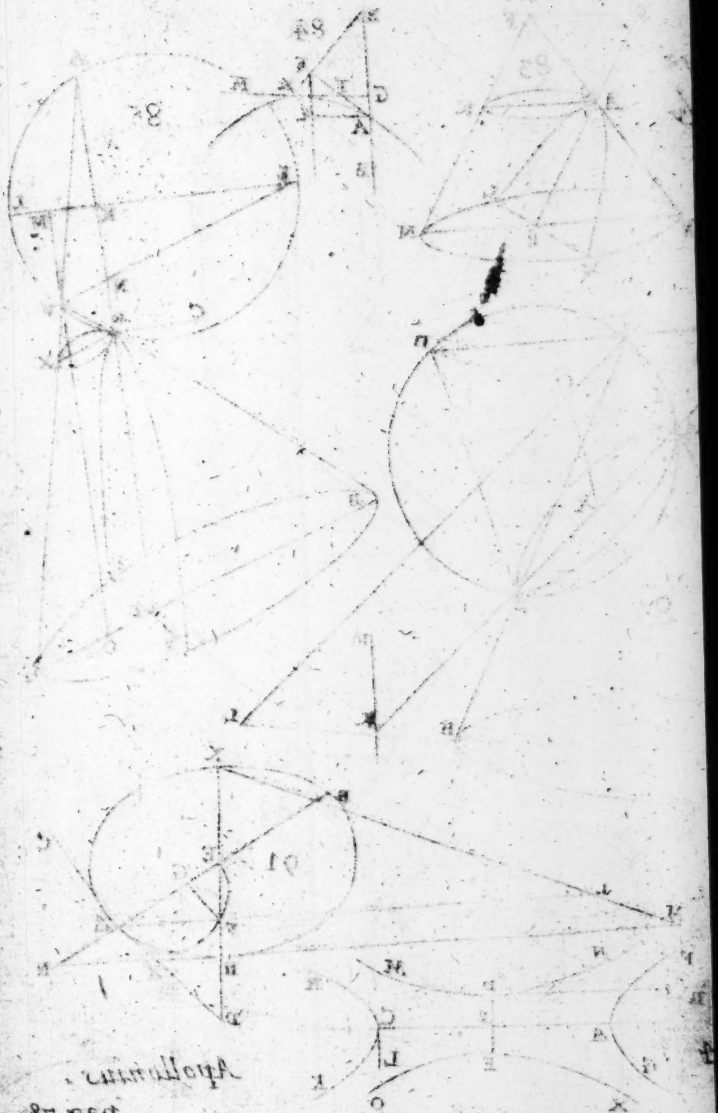

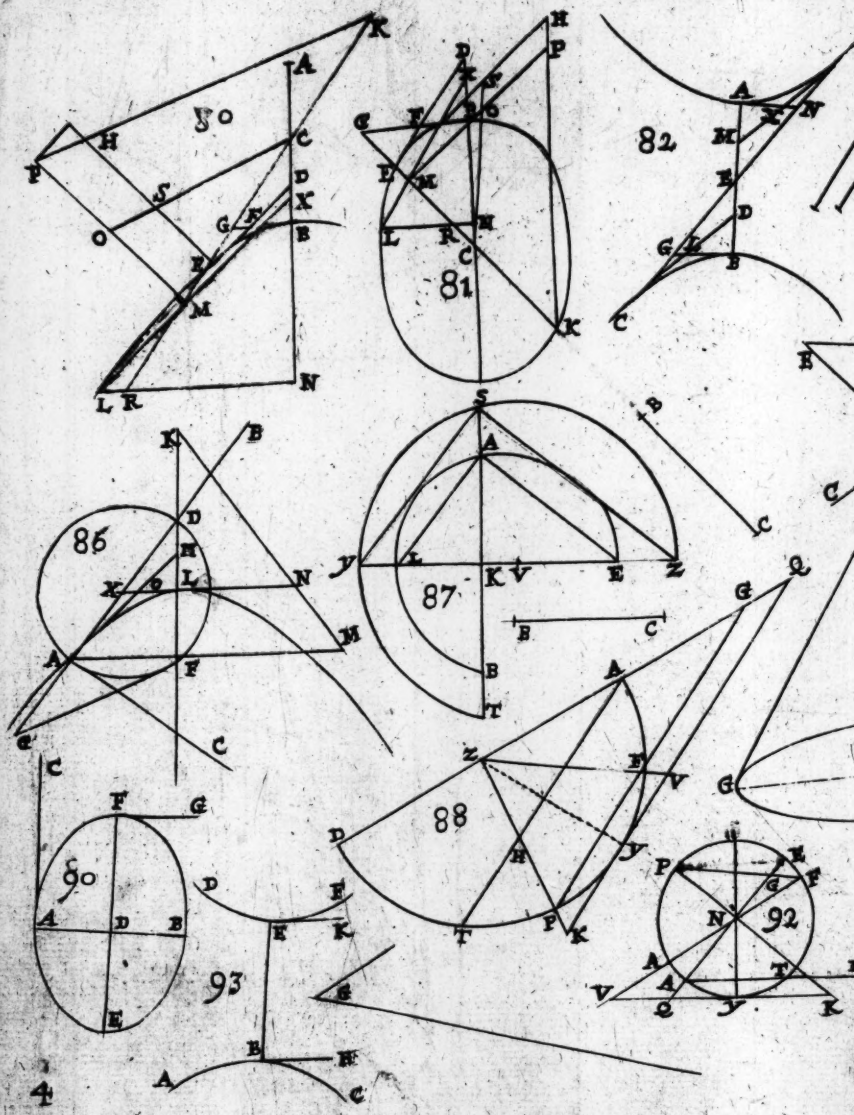
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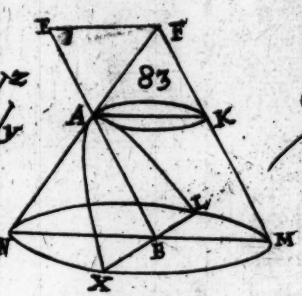
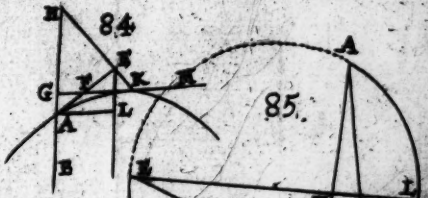
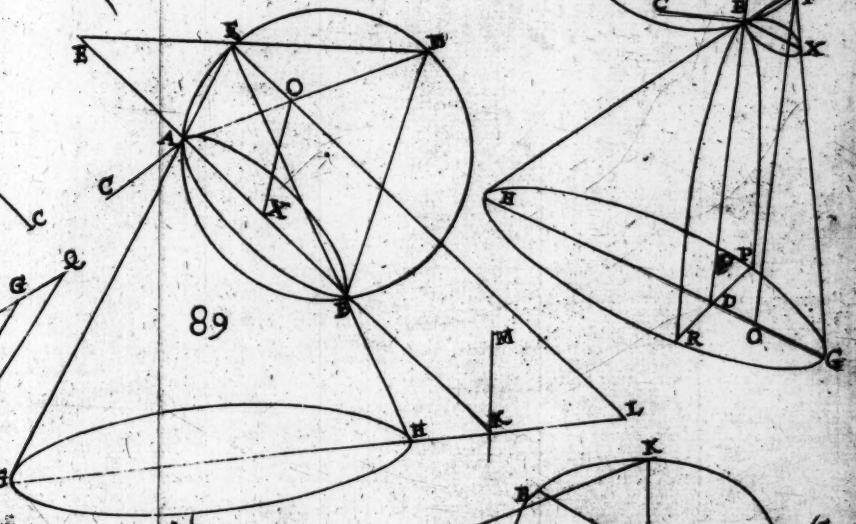
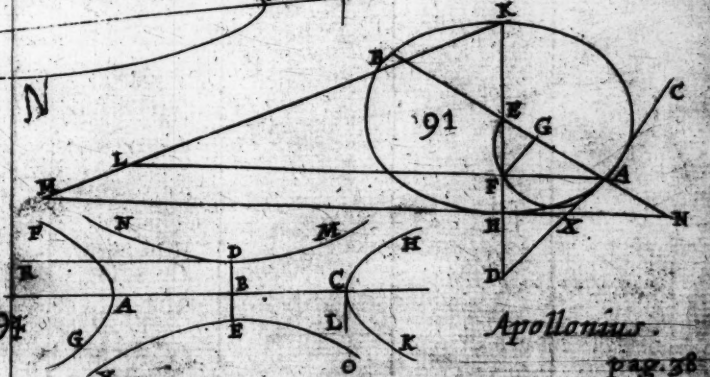
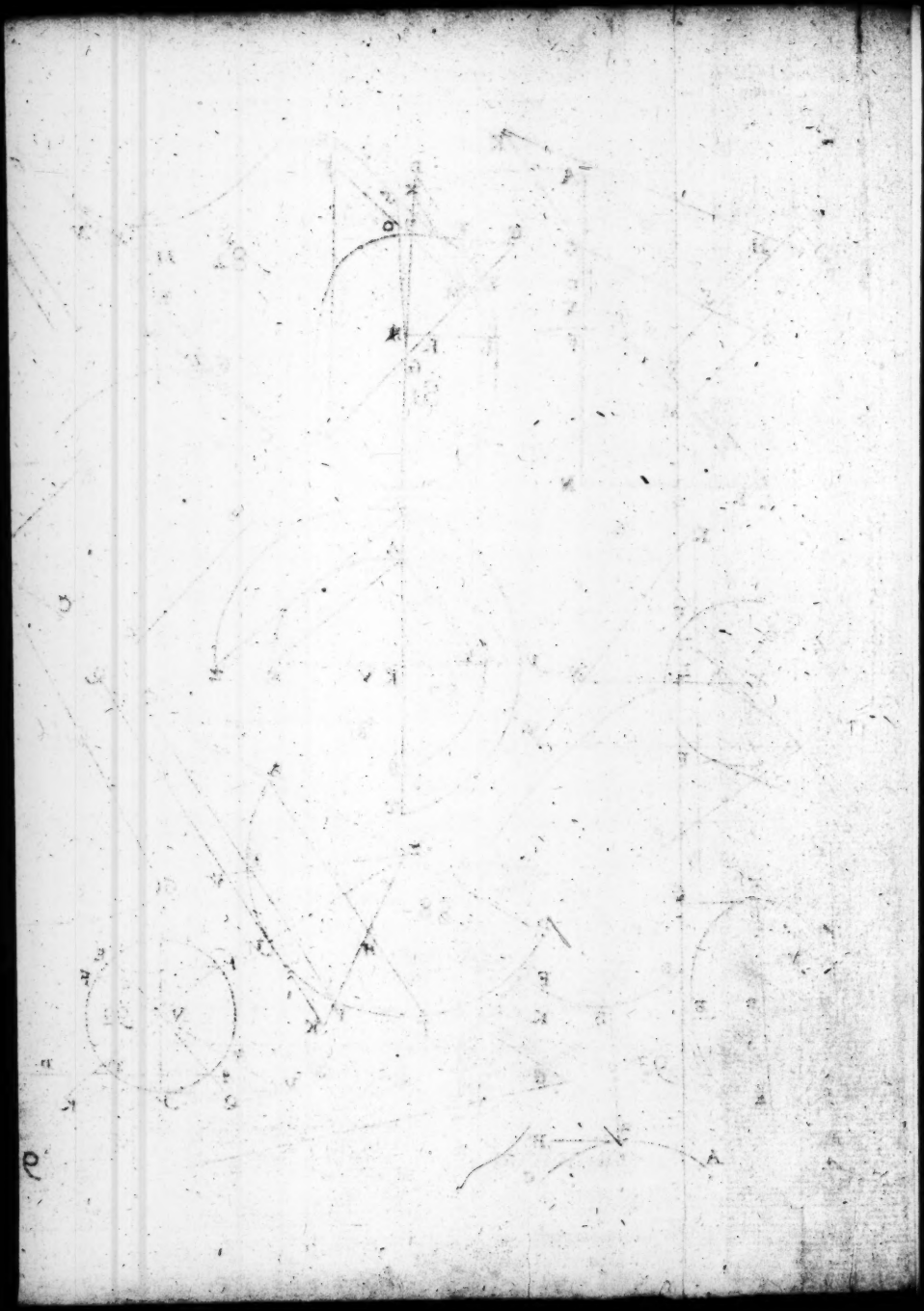








Apollonius.



APOLLONII
CONICORUM
LIB. II.

Prop. I.

SI hyperbolen contingat recta linea (D E) ad verticem (B); & ab ipſo ex utraque parte ſumatur (B D, B E) æqualis ei, quæ poteſt quartam figuræ partem, lineæ (C D, C E) quæ è ſectionis centro (C) ad ſumptos contingentis terminos (D, E) ducuntur, cum ſectione non convenient. Fig. 95.

Sumpto utcunque in C D puncto H, ordinatim applicetur H G F. Eſtque A F × F B. F Gq [a]:: (T. R [b]:: Tq. T R [c] :: $\frac{Tq.}{4}$. $\frac{TR}{4}$ [d] :: C Bq. B Dq [e] ::) C Fq. F Hq. ergo cùm A F × F B. [f] ⊐ C Fq. [g] erit F Gq ⊐ F Hq. ergo punctum H eſt extra ſectionem. Idémque de reliquis rectæ C D punctis oſtendetur. ergo tota C H eſt extra ſectionem. *Q. E. D.*

a 21. 1. *hujus.*
b 1. 6.
c 15. 5.
d *hyp.*
e 4. 6. *et* 22. 6.
f 6. 2.
g 14. 5.

Coroll. C Bq. B Dq :: T. R :: A F × F B. F Gq.

Prop. II.

Iiſdem manentibus, oſtendendum eſt, non eſſe alteram aſymptoton C K, quæ angulum D C E dividat. Fig. 96.

Per B ducatur B K ad C D parallela, occurrens ipſi C K in K; & per K ducatur H G K F L ad D E parallela. Eſtque H K [a] = D B, [b] & K L ⊏ B E. [c] unde H K × K L ⊏ D B × B E vel B D q. atqui ob C Bq. B Dq ([d] A F × F B. G Fq) [e] :: (C Fq. F Hq [f] ::) C Fq — A F × F B.

a 34. 1.
b 5. *ax.* 1.
c *ſch.* 48. 1.
d *cor.* 1. *hujus.*
e 4. 6. *et cor.* 22.
f 19. 5.

(6.

g 6 5. & 3. ax. × F B. F Hq—G Fq [g] (hoc eſt C Bq. H G × G L.), [h] eſt B Dq = h 9 5. (1. H G × G L. ergo H K × K L ⊏ H G × G L. [k] unde punctum K eſt k ſch. 5. 2. intra ſectionem. & proinde C K ſectionem intrat. *Q. E. D.*

Coroll. H G × G L = D Bq = ¼ T R.

Prop. III

Fig. 97. Si hyperbolen contingat recta linea (H K), cum utraque aſymptoton (E F, E G) conveniet; & ad tactum (B) bifariam ſecabitur; quadratum verò utriuſque ejus portionis (BF, BG) æquale erit quartæ parti figuræ, quæ ad diametrum (B D) per tactum ducitur.

Ducatur diameter B E D, & quartæ parti figuræ ad hanc æquentur a 1. *hujus.* ſingula B Hq, B Kq erunt ductæ E H, E K aſymptoti. [a] ergo hæ non differunt ab ipſis E F, E G.

Prop. IV. Probl. 1.

Fig. 98. Datis duabus rectis lineis (A B, AC) angulum (BAC) continentibus, & dato intra angulum (BAC) puncto (D), deſcribere per punctum (D) coni ſectionem (quæ hyperbole dicitur, ita ut datæ lineæ (A B, AC) ipſius aſymptoti ſint.

a 31. 1. b 4. 6. c *conſtr.* d *cor.* 4. 2. e 11. 6 *et* 17. 6. f *priùs.* g 53. *hujus.* h 1. *hujus.*

[a] Duc D F ad A B parallelam, & fac F C = F A, & produc C D B. Eſtque B C. C D [b] :: (C A. C F [c] :: 2. 1). ergo B Cq [d] = 4 C Dq = 4 B Dq. duc D A E, ita ut A E = D A. & fiat E D × G [e] = B Cq [f] = 4 B Dq. [g] Habes igitur diametrum E D, & rectum latus G hyperbolæ, cujus vertex D, [h] aſymptoti A B, A C. ergo factum.

Prop. V.

Fig. 99. Si parabolæ, vel hyperbolæ diameter (D B E) lineam quandam (A C) bifariam ſecet (in E), quæ (F G) ad diametri terminum (B) contingit ſectionem, æquidiſtans eſt lineæ (A C) bifariam ſectæ.

a 46. & 47. 1. *hujus.* b *hyp.* c 2. 6.

Si negas A C eſſe parallelam ipſi F G, ſit ei parallela C H. [a] ergo C K = K H. unde cùm ſit quoque C E [b] = E A, [c] erunt A H, E K parallelæ, contra 22. 1. hujus.

Prop.

Prop. VI.

Si ellipſis, vel circuli circumferentiæ diameter (A B) lineam quandam (C D) non per centrum tranſeuntem bifariam ſecet (in E), quæ ad diametri terminum (A) contingit ſectionem, æquidiſtans erit biſectæ lineæ (C D). Fig. 100.

Demonſtratur, ut præcedens.

Prop. VII.

Si coni ſectionem, vel circuli circumferentiam contingat recta linea (F G), & huic æquidiſtans (A C) ducátur in ſectione, & bifariam dividatur (in E); quæ à tactu (B) ad punctum (E) lineam bifariam dividens jungitur (B E), ſectionis diameter erit. Fig. 101.

Nam altera [a] nulla B H biſecabit A C; ergo non erit alia [b] diameter quàm B E. [a] 9. ax. 1. [b] 46. & 47. 1. hujus.

Prop. VIII.

Si hyperbolæ occurrat recta linea (A C) in duobus punctis (A, C); producta ex utraque parte conveniet cum aſymptotis (D E, D F); & lineæ (A E, C F), quæ ex ipſa abſciſſæ inter ſectionem & aſymptotos interjiciuntur, æquales erunt. Fig. 102.

Biſecetur A C in G, ducatúrque D G. [a] hæc diameter eſt. [b] ergo tangens per B (nempe H K) eſt ad A C parallela; ergo cùm H K *aſymptotis occurrat, [c] ſitque B H = B K,* etiam A C eiſdem occurret, [d] eritque G E = G F; [e] unde manent A E, C F æquales. Q. E. D. [a] 7. hujus. [b] 5. hujus. [c] 3. hujus. * [d] 4. 6. [e] 3. ax.

Prop. IX.

Si recta linea (C D) occurrens aſymptotis (A C, A D) ab hyperbolæ biſariam ſecetur, (in B) in uno tantum puncto ſectionem contingit. Fig. 103.

Occurrat alibi, ſi fieri poteſt, in E. ergo E C [a] = (B D [b] = ½ D C [b] =) B C. [c] Q. E. A. [a] 8. hujus. [b] hyp. [c] 9. ax. 1.

Prop. X.

Si recta linea (D F) ſectionem ſecans (in A, C) conveniat cum utraque aſymptoton (E D, E F), rectangulum contentum rectis lineis Fig. 104.

(D A, A F) quæ inter aſymptotos, & ſectionem interjiciuntur, æquale eſt quartæ parti figuræ factæ ad diametrum (H G), quam æquidiſtantes ipſi ductæ lineæ (A C) bifariam dividit.

Patet ex corollario 2dæ hujus.

Cor. A D × A F [a] = C D × C F.

a 1. ax. 1.

Prop. XI.

Fig. 105. Si utramque linearum (A E, A C) continentium angulum (E A C), qui deinceps eſt angulo (D A C) hyperbolen continenti, ſecet recta linea (E F), in uno tantùm puncto cum ſectione conveniet; & rectangulum conſtans ex iis (E G, F G), quæ interjiciuntur inter lineas A E, A C) angulum continentes, & ſectionem, æquale erit quartæ parti quadrati ex diametro (A B), quæ ſecanti lineæ (E F) æquidiſtans ducitur.

a cor. 47. 1. hujus.
b 26. 1. hujus.
c 5. hujus.
d 3. hujus.
e 23. 6.
f 4. 6.
g 10. hujus.
h 14. 5.

Ducatur A L ad E F parallela. [a] hæc diameter eſt ſectionis. [b] ergo E F in unico puncto (G) occurrit ſectioni. Per G ordinatim applicetur H G L K; [c] hæc tangenti C D (per verticem B ductæ) eſt parallela. C B q [d] (C B × B D). B A q [e] = (C B. B A [f] (H G. G F) + B D. B A [g] (G K. G E) [e] =) H G × G K. G F × G E. ergo cùm C B q [f] = H G × K G, [h] erit B A q = E G × G F. Q. E. D.

Prop. XII.

Fig. 106. Si ab aliquo puncto (D) eorum, quæ ſunt in ſectione, ad aſymptotos (B A, B C) duæ rectæ lineæ (D E, D F) in quibuſlibet angulis ducantur; & ab altero puncto (G) in ſectione ſumpto ducantur aliæ lineæ (G H, G K) his ipſis (D E, D F) æquidiſtantes, rectangulum conſtans ex æquidiſtantibus (G H, G K) æquale eſt ei, quod fit ex iis (D E, D F), quibus illæ æquidiſtantes ductæ fuerant.

a cor. 10. hujus.
b 14. 6.
c 4. 6.
d 11. 5.
e 16. 6.

Connexa G D protrahatur utrinque in A, & C. Eſtque D A × D C [a] = G A × G C: [b] unde G A. D A [c] (G H. D E) [b] :: (D C. G C [c] ::) D F. G K [d] :: G H. D E. [e] ergo G H × G K = D F × D E. Q. E. D.

Prop. XIII.

Fig. 107. Si in loco aſymptotis (A B, A C) & ſectione terminato, quædam recta linea (E F) ducatur, æquidiſtans aſymptoton alteri (A B), in uno tantùm puncto cum ſectione conveniet.

Si

Si primò negas E F sectioni occurrere, per G (punctum utcunque sumptum in sectione) ducantur G C, G H ad A B, A C parallelæ. Sitque A E * E F [a] = G C * G H; connexa A F [b] sectioni occurret, puta in K. Ex quo ducantur K L, K D ipsis A B, A C itidem parallelæ. ergo K L * (A L) K D [c] = (G H * G C [d] =) A E * E F. [e] Q.E.A.

[a] 12.6. et 16.6. [b] 2. hujus. [c] 12. hujus. [d] constr. [e] 9. ax. [f] 12. hujus. [g] 34. 1. & sch. [h] 9. ax. (48. 1.

Proinde E F sectioni occurret, nempe in M. Dic alibi occurrere, puta in N. ducantúrque M X, N B ad A C parallelæ. ergo E M * M X [f] = (E N * N B [g] =) E N * M X. [h] Q.E.A.

Prop. XIV.

Asymptoti (A B, A C), & sectio in infinitum productæ ad seipsas propius accedunt; & ad intervallum perveniunt minus quolibet dato intervallo (K). Fig. 108.

Ducantur E H F, C G D tangenti utcunque parallelæ; pérque A, & occursum H jungatur A H X. Estque C G * G D [a] = E H * H F. [b] quare G D. H F :: E H. C G. ergo cùm G D ⊏ (X D ⊏) H F, [c] erit E H ⊏ C G; paritérque omnes decrescunt versùs partes C G. Sumatur E L ⊐ K, ducatúrque L N ad A C parallelæ; [d] hæc sectioni occurret, puta in N, per quod ducatur M N B ad F F parallela. Estque M N [e] (= E L) [f] ⊐ K, Q.E.D,

[a] 10. hujus. & 1. ax. [b] 14. 6. [c] 14. 5. [d] 13. hujus. [e] 34. 1. [f] constr.

Cor.

Ex hoc manifestum est lineas A B, A C ad sectionem accedere propiùs, quàm omnes aliæ asymptoti (quales A Y, A Z); & angulum B A C minorem esse quolibet angulo, qui aliis ejusmodi lineis continetur.

Prop. XV.

Oppositarum sectionum (A, B) asymptoti communes sunt. Fig. 109.

Sint A B diameter, C centrum, ac D E, F G coutingant sectiones in A, B; è quibus utrinque abscindantur A D, A E, B F, B G, ut singularum quadrata æquentur quartæ parti [a] figuræ ad A B: itaque junctæ C D, C E sectionis A, & C F, C G, sectionis B [b] asymptoti erunt. quoniam verò utraque D E, F G ~~ordinatim applicatis ad A B~~ ~~sint parallelæ~~; ~~& [c] proinde~~ sibi invicem ~~istæ~~ parallelæ sunt, [d] erit ang. B A C = G B C; paréque [e] sunt A C, B C; [f] & A D, B G. [g] ergo ang. A C D = ang. B C G. [h] ergo D C G est recta linea. Similiterque

[a] 14. 1. hujus. [b] 1. hujus. [c] 30. 1. [d] 29. 1. [e] hyp. [f] constr. [g] 4. 1. [h] cor. 15. 1.

✕ Cor. 44. 1. hujus.

perque E C F recta est. Unde patet propositum.

Coroll. Tangentes D E, F G parallelæ sunt sibi invicem.

Prop. XVI.

Fig. 110. Si in oppositis sectionibus (A, B), ducatur quædam recta linea (H K), secans utramque linearum (C D, C F) continentium angulum (D C F), qui deinceps est angulo (D C E, vel G C F) sectiones continenti; cum utraque oppositarum in uno tantùm puncto conveniet, & lineæ (H L, K M), quæ ex ipsa abscissæ inter asymptotos (C D, C F), & sectiones interjiciuntur, æquales erunt.

a 11. *hujus.* Quod H K sectionibus occurrat, [a] manifestum est; occurrat punctis L, M; perque centrum C ducatur A B ad L M parallela. Estque
b 29. 1. *hujus.* K L * L H [a] = (A Cq [b] = B Cq [a] =) H M * M K. [c] ergo K L.
c 16. 6. M K :: H M. L H. & componendo L M. M K :: M L. L H. [d] unde
d 9. 5. M K = L H, *Q. E. D.*

Prop. XVII.

Fig. 111. Oppositarum sectionum (A, B; C, D) quæ conjugatæ appellantur, asymptoti communes sunt.

a *cor.* 15. *hujus.* Sint A B, C D conjugatæ diametri sectionum; perque vertices A,
b *def. ad* 16. & 17. 6. B, C, D ducantur tangentes F G, K H, F K, G H; [a] Liquet, F G H K esse parallelogrammum, & diagonales F H, G K esse asymptotos; nam
c *cor.* 4. 2. figuris ad A B [b] æquatur C Dq, [c] hujusque quartæ parti singula A Fq,
d 1. & 16. *hujus.* A Gq, B Hq, B Kq æquantur. [d] unde F H, G H sunt asymptoti sectionum A, B. pariterque hæ asymptoti sunt sectionum C, D. quare constat propositum.

Prop. XVIII.

Fig. 112. Si uni (C) oppositarum sectionum, (A, B; C, D) quæ conjugatæ dicuntur, occurrat recta linea (E F), & producta ad utrasque partes extra sectionem cadat; cum utraque (A, B) sectionum, quæ deinceps sunt, in uno tantùm puncto conveniet.

a 13. *hujus.* Sint G H, L K asymptoti, [a] his occurrit E F. [b] ergo liquet pro-
b 16. *hujus.* positum.

Prop.

Prop. XIX.

Si in oppositis sectionibus (A, B, C, D) quæ conjugatæ appellantur, ducatur recta linea (E F), ipsarum quamvis (C) contingeus cum sectionibus (A, B), quæ deinceps sunt, * conveniet (in G, H); & ad tactum (C) bifariam secabitur. Fig. 113. * per præced.

Sint K L. M N asymptoti. Et ob C E [a] = C F, & E G [b] = F H, [c] erit C G = C H. Q. E. D. a 3. hujus. b 16. hujus. c 3. ax. 1.

Prop. XX.

Si oppositarum sectionum (A, B, C, D) quæ conjugatæ appellantur, unam (A) contingat recta linea (E F); & per ipsarum centrum (X) ducantur duæ lineæ; una quidem (E X) per tactum, altera verò (X G) contingenti (E F) æquidistans; quousque occurrat (in G) uni (C) earum sectionum, quæ deinceps sunt; recta linea (G H) quæ in occursu (G) sectionem contingit, æquidistans erit lineæ (E X) per tactum, & centrum ductæ; quæ verò (E Z, G O) per tactus, & centrum ducentur, oppositarum sectionum conjugatæ diametri erunt. Fig. 114.

Sint A M, C N [a] rectæ sectionum latera, & per puncta E, G, C ordinatim applicentur E K, G L, C R P. Estque X K. K E. + F K. K E [b] = (X K * F K. K E q [c] = B A. A M [d] :: N C. C D :: [e] G Lq. L X * L H [b] =) G L. L X + G L. L H. atqui (ob L X ad E K, & L G ad X K, & G X ad E F parallelas) [e] est F K. K E :: G L. L X. ergo manet X K. K E :: G L. L H. [f] ergo trigona E K X, H L G similia sunt. [f] & ang. E X K = H G L. itemque totus ang. G X K [g] = X G L. [h] ergo manet ang. E X G = H G X. [k] quare rectæ E X, G H parallelæ sunt. a 14. 1. hujus. b 23. 6. c 37. 1. hujus. d vid. Mos. e 4. 6. f 6. 6. g 29. 1. h 3. ax 1. k 27. 1.

Porro, fiat P G. G R [l] :: H G. S. [m] est ergo 2 S, juxta quam possunt ordinatæ ad diametrum G O. item T X * K E (* X V) [n] = C Xq; (o vel T X, C X, K E ÷÷) [p] quare T X. K E (c hoc est T F. F E, q vel triang. T X F. F X E) :: (T Xq. C Xq. [p] ::) triang. T X F. X C P. ergo triang. F X E [r] = (X C P [s] =) H X G. item ang. X E F [s] = X G H. ergo reciprocè G H. E X [t] :: E F. G X. [u] unde G H * G X [r] = E X * E F. ergo S * G X. E X * E F [x] :: (S. * G X. G H * G X [s] :: S. G H [z] :: G R. G P [y] :: E X. E F [y] ::) E Xq. E X * E F. ergo E Xq l 12. 6. m 5 1. 1. hujus. n 38. 1 hujus. o 17. 6. p cor. 20. 6. q 1. 6. r 9. 5. s sch. 43. huj. t 15. 6. u 16. 6. x 7. 5. y 2. 6. z constr. et inversè a 4. 6.

b =

b 5. 5. c *prius.* d *cor.* 4. 2. e 6. *ax.* 1. f *def. ad* 16. 1. *hujus.*

[b] = (S × GX [c] =) $\frac{1}{4}$ fig. ad GO. quare EZq [d] (4EXq) [e] = fig. ad GO. Simili discursu erit GOq figuræ ad EZ æquale. quare EZ, GO [f] sunt conjugatæ diametri. *Q. E. D.*

Not. Quod BA. AM :: NC. CD, sic patet. Quoniam NC. BA :: BA. CD & BA. CD :: CD. AM. erit ex æquali NC. CD :: BA. AM.

Cor. BA. AM :: NC. CD.

Prop. XXI.

Fig. 115. Iisdem positis, ostendendum est, punctum (E), in quo contingentes lineæ (AE, CE) conveniunt, ad unam asymptoton esse.

a 34. 1. b 3. *hujus.* c *hyp.* d 1. *hujus.*

Nam quia CXq (a vel AEq) [b] æquatur $\frac{1}{4}$ CDq, [c] hoc est quartæ parti figuræ ad AB, [d] erit XE asymptotos.

Prop. XXII.

Fig. 116. Si in oppositis sectionibus (A, B, C, D) quæ Conjugatæ appellantur, ex centro (X) ad quamvis sectionem (C) ducatur recta linea (XC) & huic æquidistans ducatur altera (EKLH) quæ cum una (A) ex sectionibus, quæ deinceps sunt, & cum asymptotis (XE, XF) conveniat; rectangulum constans ex portionibus (EK, KH) lineæ ductæ inter sectionem, & asymptotos interjectis, quadrato lineæ (XC) quæ ex centro ducitur, æquale erit.

a 5. *hujus.* b 48. 1. *hujus.* c 20. *hujus.* d 10. *hujus.* e *cor.* 4. 2.

Bisecetur KL in M, ducaturque (per M, & X) recta MAXB; Igitur tangenti ad A [a] parallela est EH; eademque proinde ad AB diametrum [b] ordinatim applicatur; & AB, CD [c] sunt conjugatæ diametri; quare EK × KH [d] = ($\frac{1}{4}$ TR [c] =) CXq.

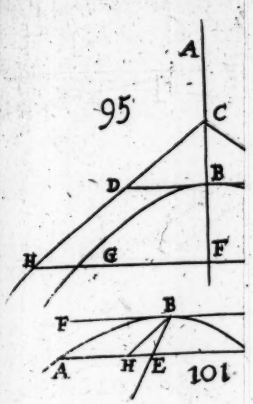
Cor. EK × KH = CXq.

Prop. XXIII.

Fig. 117. Si in oppositis sectionibus A, B, C, D) quæ conjugatæ appellantur, ex centro (X) ducatur quædam recta linea (XC) ad quamvis sectionem (C); & huic æquidistans (KL) ducatur, quæ cum tribus, quæ deinceps sunt, sectionibus (C, A, D) conveniat, rectangulum constans ex portionibus (KM, ML) lineæ ductæ (KL) inter tres sectiones interjectis, duplum erit quadrati ejus lineæ (XC), quæ ex centro ducitur.

a *cor.* 22. *huj.* b 11. *hujus.* c 1. *ax.* 1.

Sint EF, GH sectionum asymptoti. ergo HM × ME [a] = (CXq [b] =) HK × KE. [c] quare 2 CXq = (HM × ME + HK × KE. =) LM,



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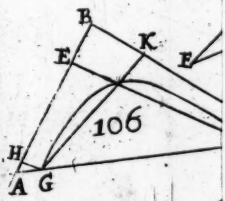

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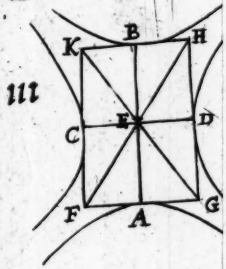

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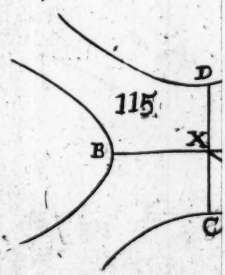

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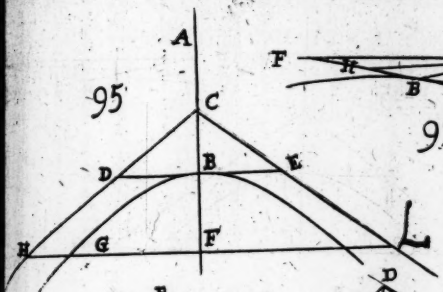
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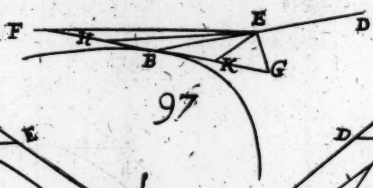
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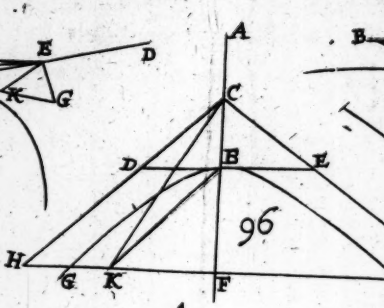
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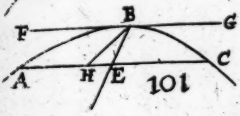
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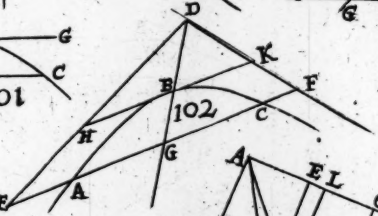
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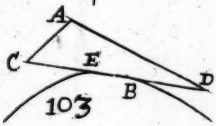
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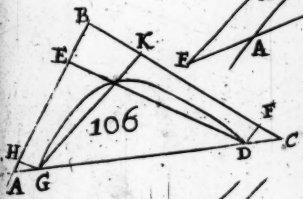
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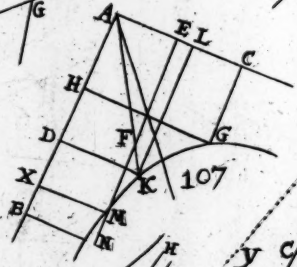
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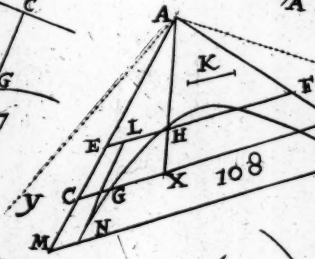
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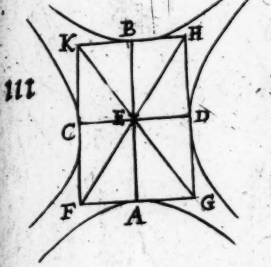
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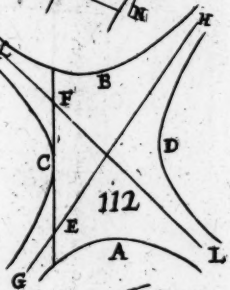
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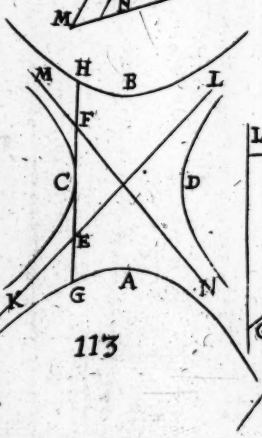
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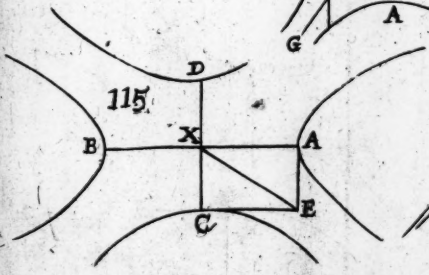
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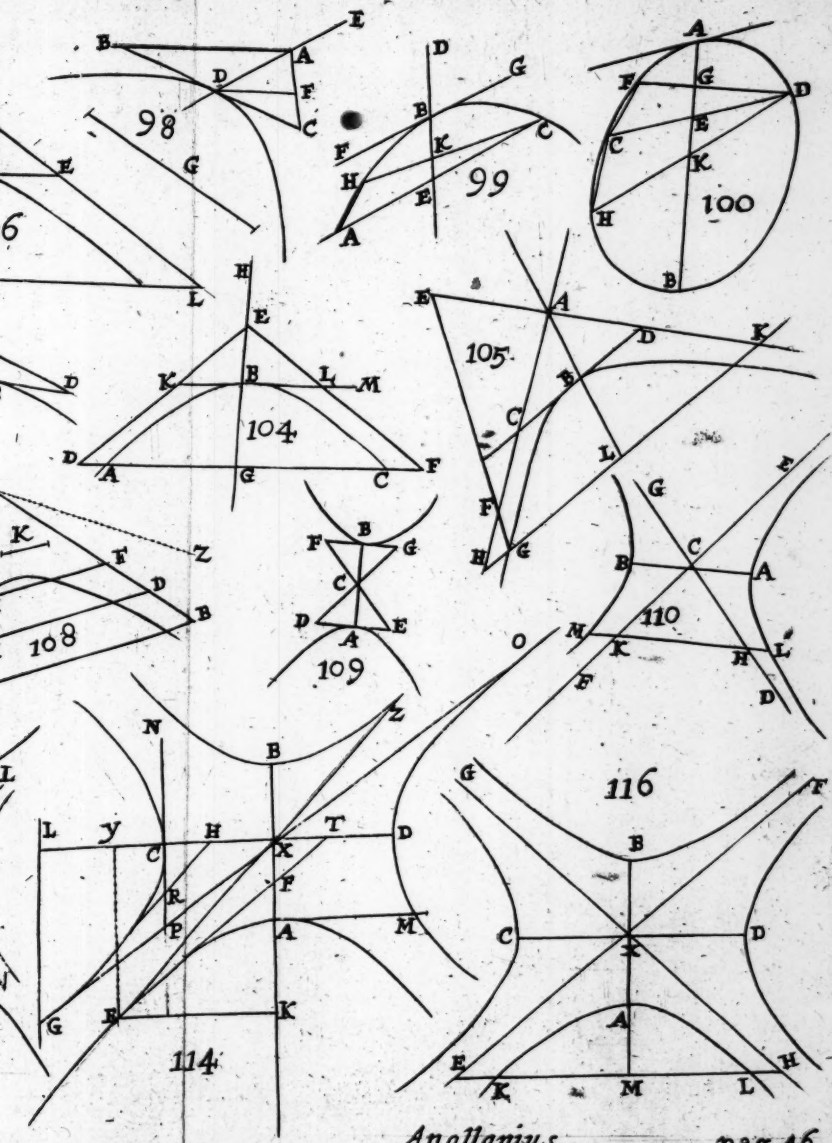
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LM × MK. (Nam LM × MK [b] = LH × MK [c] (KE × MK) + HM × MK [d] = KE × MK + HM × ME + KE × HM [e] = KE × HK + HM × ME.)

[d] 1. 6.
[e] 16. *hujus.*

Prop. XXIV.

Si parabolæ occurrant duæ rectæ (AB, DC), utraque in duobus punctis, & nullius ipsarum occursus, alterius occursibus contineatur, convenient inter se extra sectionem. Fig. 118.

Per B, C puncta ductæ sint diametri EF, GH. [a] Hæ parallelæ sunt, [b] nec alibi occurrunt sectioni, unde junctâ BC, erunt anguli ABC, DCB simul duobus rectis majores. [c] ergo AB, DC extra sectionem concurrent ad partes EG.

[a] cor. 46. 1. *huj.*
[b] 26. 1. *hujus.*
[c] 13. *ax.* 1.

Prop. XXV.

Si hyperbolæ occurrant duæ rectæ lineæ (EF, GH), utraque in duobus punctis, nullius autem ipsarum occursus alterius occursibus contineatur, convenient quidem inter sese extra sectionem, sed tamen intra angulum (BAC) qui hyperbolen continet. Fig. 119.

Ducantur AF, AH, & connectatur FH. & quoniam EF, GH secant angulos AFH, AHF, concurrent intra angulum FAH, & propterea magis intra angulum BAC. Idem discursus valet, si utraq; EF, GH sectionem contingunt; aut si una contingat, altera duobus punctis secet. *vid. cor.* 31. 1. *hujus.*

Prop. XXVI.

Si in ellipsi, vel circuli circumferentia, duæ rectæ lineæ (CD, EF) non transeuntes per centrum (H), se invicem secent (in G), bifariam sese non secabunt. Fig. 120.

Per G ducatur diameter AB; suntque omnes, quas AB bisecat, tangenti ad A [a] parallelæ, & proinde sibi invicem parallelæ. Ergo CD, EF non bisecantur in G. *Q. E. D.*

[a] 6. *hujus.*

Prop. XXVII.

Si ellipsim, vel circuli circumferentiam contingant duæ rectæ lineæ (CD, EF; & siquidem ea (AB), quæ tactus (A, B) conjungit, per centrum Fig. 121. 122.

centrum tranſeat ſectionis, contingentes lineæ (C D, E F) ſibi ipſis æquidiſtabunt; ſin minùs, convenient inter ſeſe ad eaſdem partes centri.

a 6. *hujus.* b 30. 1. c *ex priore parte hujus.* d 29. 1. e 13. *ax.* 1.

Si A B per centrum tranſit, [a] erit utraque C D, E F ordinatim applicatis parallela, [b] ergo ſibi invicem. Sin A B non tranſeat, per centrum, ducatur diameter A H, & per H tangat K L. [c] ergo C D, K L parallelæ ſunt: [d] quare anguli B A H, K H A duobus rectis minores ſunt. [e] ergo E F, H K convenient ad partes B H: & proinde E F, A C convenient ad partes A B. *Q. E. D.*

Converſè, ſi C D, K H æquidiſtent, tranſibit recta quæ tactus connectit per centrum.

Prop. XXVIII.

Fig. 123.

Si in coni ſectione, vel circuli circumferentia duas lineas æquidiſtantes (A B, C D) bifariam ſecet recta linea (F E) diameter erit ſectionis.

Sola enim F E biſecat parallelas ad A B.

a 5. & 6. *huj.* b 46. *et* 47. 1. *hujus.*

Si fieri poteſt, ſit alia F G diameter. [a] ergo quæ tangit ſectionem in G, eſt ubique A B, C D parallela. unde C H [b] = ($\frac{1}{2}$ C D =) C E. *Q. E. A.*

Prop. XXIX.

Fig. 124. 125.

Si coni ſectionem, vel circuli circumferentiam contingentes duæ rectæ lineæ (B A, C A) in idem punctum (A) conveniant; & ab eo, ad punctum (D), quod lineam (B C) tactus (B, C) conjungentem bifariam ſecat, ducatur alia linea (A D), ſectionis erit diameter.

a 5. *hujus.* & 47. 1. *huj.* b *hypoth.* c * d 1. *ax.* e 9. *ax.*

Si fieri poteſt, ſit alia D E diameter; jungatúrque C E, *ſectioni occurrens in F, per quod ducatur F H K G ad C B parallela. ergo F H [a] = H K, item (ob C D [b] = D B) [c] eſt F H = H G. [d] ergo H K = H G.* *Q. E. A.*

Prop. XXX.

Fig. 126.

Si coni ſectionem, vel circuli circumferentiam contingentes duæ rectæ lineæ (B A, C A) in unum punctum (A) conveniant, diameter (A D), quæ ab eo puncto ducitur lineam (B C) tactus (B, C) conjungentem bifariam ſecabit.

a 29. *hujus.* b *cor.* 46. 1. *huj.*

Si fieri poteſt ſit B E = E C; ducatúrque A E. [a] ergo A E eſt quoque diameter ſectionis. [b] ergo in parabola A E, A D ſunt parallelæ;

læ; in reliquis sectionibus A [c] est centrum. Quæ sunt absurda. c vid. 47. 1. hujus.

Prop. XXXI.

Si utramque oppositarum sectionum (A, B) contingant duæ rectæ lineæ (C D, E F), siquidem ea (A B), quæ tactus (A, B) conjungit, per centrum transeat, contingentes lineæ (C D, E F), æquidistantes erunt; sin minus, convenient inter sese ad easdem partes centri Fig. 127. 128.

Probatur, ut 27ma hujus. et per cor: 44 5tis hujus.

Prop. XXXII.

Si utrique oppositarum sectionum occurrant rectæ lineæ (A B, C D), ipsas vel in uno puncto contingentes, vel in duobus secantes, & productæ inter se conveniant; punctum in quo conveniunt, erit in angulo (K L G), qui deinceps est angulo (G L H, vel F L K) sectiones continenti. Fig. 129.

Sint F G, H K sectionum asymptoti; hisce [a] occurret utraque A B, C D. productæ igitur occurrent sibi invicem sub angulo H L F, vel G L K, prout inclinantur ad has, aut illas partes. a 8. hujus.

Prop. XXXIII.

Si oppositarum sectionum (A, B) uni (A) occurrens recta linea (C D) ex utrâque parte extra sectionem cadat; cum altera sectione (B) non conveniet; sed transibit per tres locos, quorum unus quidem est sub angulo sectionem continente, duo verò sub iis angulis, qui deinceps sunt. Fig. 130.

Liquet C D [a] occurrere asymptotis duobus punctis; ergo non alibi; ergo non alteri sectioni, quam semper complectuntur asymptoti. a 8. hujus.

Prop. XXXIV.

Si oppositarum sectionum unam (A) contingat recta linea (CD); & huic ducatur æquidistans (E F) in altera sectione (B); quæ à tactu (A) ad (G) medium lineæ æquidistantis (E F) ducitur (A G), oppositarum sectionum diameter erit. Fig. 131. cor: 44. 1. hujus

Si fieri potest, sit altera A K diameter. ergo tangens per occursum H parallela est ad C D, ideóque ad E F. [b] quare E K [b] = (K F = $\frac{1}{2}$ EF [c] =) E G. [d] Q. E. A. a 5 hujus. b 47. 1. hujus. c hyp. d 9. ax. 1.

Prop. XXXV.

Fig. 132. Si diameter (A B) in oppoſitarum ſectionum una (B) rectam lineam (C D) bifariam ſecet (in E); quæ in diametri termino (A) contingit alteram ſectionem (A), lineæ biſectæ (C D) erit æquidiſtans.

a 48. 1. *hujus.*
b *hyp.*
c 2. 6.
d 22. 1. *huj.*

Si fieri poteſt, ſit altera D F tangenti parallela. ergo D G [a] = G F. Item D E [b] = E C. [c] ergo C F ad E G eſt parallela. [d] *Q. E. A.*

Prop. XXXVI.

Fig. 133. Si in utraque oppoſitarum ſectionum (A, B) ducantur rectæ lineæ (C D, E F) inter ſe æquidiſtantes, quæ (G H) ipſarum medium conjungit, oppoſitarum ſectionum diameter erit.

a 5. *hujus.*
b 30. 1.
c 48. 1. *hujus.*
d *hyp.*
e 9. *ax.* 1.

Si fieri poteſt, ſit altera G K diameter: [a] ergo tangens per A ad C D parallela eſt; [b] adeóque ad E F. unde E K ([c] = K F = ½ E F [d] =) E H. [e] *Q. E. A.*

Prop. XXXVII.

Fig. 134. Si oppoſitas ſectiones (A, B) ſecet recta linea (C D), non tranſiens per centrum X; quæ (E X) ab ipſius medio (E) ad centrum ducitur, oppoſitarum ſectionum diameter erit, quæ recta appellatur: tranſverſa verò diameter, ipſi conjugata, eſt ea (A B), quæ per centrum ducitur æquidiſtans lineæ biſectæ (C D).

a 30. 1. *hujus.*
b *hyp.*
c 2. 6.
d 4. 6.
e 34. 1.
f 6. *hujus.*
g 16. 1. *hujus.*

Ducatur D X ſectioni [a] occurrens in F, & connectatur F C, & producatur B A G. Atque ob C E [b] = E D; & F X [a] = X D [c] erit F C ad X E parallela, & F G [d] = (X E [e] =) G C. [f] quare F C tangenti ad A parallela eſt. [g] ergo A B, & E X ſunt conjugatæ diametri. *Q. E. D.*

Prop. XXXVIII.

Fig. 135. Si oppoſitas ſectiones (A, B) contingant duæ rectæ lineæ (C X, D X), convenientes in uno puncto (X); quæ (X E) ab eo puncto ad medium (E) lineæ (C D) tactus (C, D) conjungentis ducitur, oppoſitarum ſectionum diameter erit, quæ recta vocatur, tranſverſa verò ipſi conjugata (A B), quæ per centrum ducitur, æquidiſtans lineæ (C D) tactus conjungenti.

Si

Si fieri poteſt, ſit altera E F recta diameter, cui occurrat D X (producta in F ; & connectatur C F, [a] ſectioni occurrens in A ; per quod ducatur A B ad C D parallela. ergo A G [b] = G B. item (ob C E [c] = E D) eſt A G [d] = G K. [e] unde G B = G K. [f] Q. E. A.

a 32. 1 *hujus.*
b 12. *def.* 1 *hujus.*
c *hyp.*
d *cor.* 2. 6.
e 1. *ax. f.* 9. *ax.*

Prop. XXXIX.

Si oppoſitas ſectiones (A, B) contingant duæ rectæ lineæ (C E, D E) in unum punctum (E) convenientes ; quæ (E F) per punctum illud (E), & centrum ducitur, lineam tactus (C, D) conjungentem bifariam ſecabit. Fig. 136.

Si fieri poteſt, ſit C G = G D. [a] ergo ducta G E erit diameter : [b] atqui F E eſt diameter. ergo interſectio E eſt centrum ſectionis. [c] Q. E. A.

a 38. *hujus.*
b *hyp.* 37.
c 32. *hujus.*

Prop. XL.

Si oppoſitas ſectiones (A, B) contingentes duæ rectæ lineæ (C E, E D) in unum conveniant ; & per punctum (E), in quo conveniunt, ducatur linea (F G) æquidiſtans tactus conjungenti (C D), & ſectionibus occurrens (in F, G) ; quæ (F H, G H) ab occurſibus ad medium (H) lineæ (C D) tactus conjungenti ducuntur, ſectiones ipſas contingunt. Fig. 137.

Ducatur E H, [a] erit hæc recta diameter ; & tranſverſa A B, ducta per centrum X ad C D parallela : unde E X × X H [b] æquatur quartæ parti figuræ ad A B. Hinc, cùm F E [c] ſit ordinatim applicata, [d] liquet F H tangere ſectionem A. Pari modo G H ſectionem B continget. Q. E. D.

a 37. *hujus.*
b 38. 1 *hujus.*
c 38. *hujus.*
d 38. 1. *hujus.*

Prop. XLI.

Si in oppoſitis ſectionibus duæ rectæ lineæ (A D, C B), ſe invicem ſecent, (in E) non tranſeuntes per centrum (X), ſeſe bifariam non ſecabunt. Fig. 138.

Ducatur E X ; ergo ſi A D, C B ſe mutuò biſecent in E, [a] erit E X diameter conjugata illi X F, quæ per X ducitur ad C B parallela ; eritq; E X * tangenti ad F parallela. Paritérque (ductâ X H ad D A parallelâ) erit E X tangenti ad H parallela. [c] Ergo tangentes ad F, H ſibi invicem parallelæ ſunt. [c] Q. E. A.

a 37. *hujus.*
* *vid.* 37. *hujus.*
c 31. *hujus.*

Prop. XLII.

Fig. 139. Si in oppoſitis ſectionibus (A, B; C, D) quæ conjugatæ appellantur, duæ rectæ lineæ (E F, G H) ſe invicem ſecent, non tranſeuntes per centrum (X); bifariam ſeſe non ſecabunt.

Per centrum X ducantur A B ad E F, & C D ad G H parallelæ, & connectatur X K: poſito igitur ipſas E F, G H ſe mutuò biſecare, [a] erunt X K, A B, & X K, C D conjugatæ diametri, unde tangens per A tangenti per C erit parallela (utpote utraque ipſi X K) quod fieri nequit: [b] conveniunt enim hæ ad unam aſymptoton. Ergo E F, G H ſe mutuò non biſecant.

a 37. *hujus.* * 5. *hujus.* b 21. *hujus.*

Prop. XLIII.

Fig. 140. Si unam (A) oppoſitarum ſectionum (A, B; C D) quæ conjugatæ appellantur, ſecet recta linea (E F) in duobus punctis (E, F); & a centro (X) ducantur duæ lineæ (X G, X C), una quidem (X G) ad medium (G) lineæ ſecantis (E F), altera verò (X C) ipſi (E F) æquidiſtans, erunt hæ (X G, X C) oppoſitarum ſectionum conjugatæ diametri.

Nam quia tangens in A ad E F [a] parallela eſt; & [b] proinde ad C X, [c] erunt A X, C X conjugatæ diametri.

a 5. *hujus.* b *hyp.* & 30. 1. c 20. *hujus.*

Prop. XLIV. Probl. 2.

Fig. 141. Datâ coni ſectione (A C E), diametrum invenire.

Analyſis. Factum ſit; & ſit C H diameter; & ad hanc ordinatim applicentur A E, B D: biſecat has diameter C H in H, & F.

Compoſitio. Ducatur utcunque recta A E ſectionem ſecans punctis A, E; & huic parallela fiat B D; biſecentur hæ in H & F; & connectatur H F C. [a] Erit H C diameter ſectionis. Eodem licet modo infinitas diametros invenire.

a 28. *hujus.*

Prop. XLV. Probl. 3.

Fig. 142. 143. Datâ ellipſi, vel hyperbola centrum invenire.

[a] Duæ ducantur utcunque ſectionis diametri A B, C D; erit harum interſectio centrum ſectionis.

a 44. *hujus.*

Prop.

Prop. XLVI. Probl. 4.

Datæ parabolæ (F C E) axem invenire. Fig. 144.

Analyſis. Sit C D axis, eíque perpendicularis F E; eſt ergo FD = D E. Quòd ſi ducatur utcunque diameter AB, [a]erit hæc ipſi CD parallela; atque idcirco ipſi F E perpendicularis. Hinc

[a] 46. 1. *hujus.*

Componitur ſic. [a] Ducatur utcunque diameter A B, eíque [b] ſtatuatur perpendicularis E F; biſectaque E F in D, [b] erigatur perpendicularis D C; erit hæc axis parabolæ: eſt enim D C diameter, [c]quia parallela diametro A B; & biſecat ipſi perpendiculares E F, (neque enim ulla alia ipſas biſecabit). [c]ergo eſt axis.

[a] 44. *hujus.* [b] 11. 1. [c] 13. *def.* 1. *hujus.*

Prop. XLVII. Probl. 5.

Datæ hyperbolæ, vel ellipſis (A B C) axes invenire. Fig. 145. 146.

Analyſis.

Eſto K D axis; ergo [a] biſecat hæc ſibi perpendiculares utcunque ductas A C, in D. Itaque ſi è K [b] centro ſectionis connectantur K A, K C, [c] erunt K A, K C æquales. Hinc

[a] 18. *def.* 1. *hujus.* [b] 45. *hujus.* [c] 4. 1.

Compoſitio.

[d] Sume K centrum ſectionis, & centro K duc utcunque circulum A E C, ſectioni occurrentem punctis A, C, quæ connectat recta AC; biſecetur autem A C in D, & connectatur DK. Erit DK axis. Fig. 147. 148.

[d] 45. *hujus.*

Nam ductis K A, K C, [e] liquet trigona K D A, K D C ſibi mutuò eſſe æquilatera, & [f] proinde æquari angulos K D A, K D G; ac idcirco [g] rectos eſſe: [g] unde K D eſt axis.

[e] *conſtr.* [f] 8. 1. [g] 18. *def.* 1. *huj.*

Quòd ſi per K ducatur MN ipſi AC parallela, [h] erit M N axis conjugatus ipſi K D.

[h] 16. 1. *hujus.* [i] 19. *def.* 1. *hu.*

Prop. XLVIII.

His autem demonſtratis, ſupereſt ut oſtendamus non eſſe alios axes ipſarum ſectionum.

Si fieri poteſt, ſit alius axis K G. ergo ductâ ad hanc perpendiculari A H, [a] erit L H = A H; adeóque (junctâ K L) K L [b] = (K A [c] =) K C: ideóque circulus A E C etiam tranſit per L, quod in hyperbola manifeſtè abſurdum; in ellipſi verò, ducantur C R, L S ad M N perpendiculares. Et propter L Sq + S Kq [d] = (L Kq [e] = C Kq [d] =) C Rq + R Kq. & M S × S N + S Kq [f] = (K Mq [f] =) M R × R N, + R

[a] 28. *def.* 1. *huj.* [b] 4. 1. [c] *conſtr.* [d] 47. 1. [e] *prius.* [f] 5. 2.

h 3. ax. 1. ┼R Kq. erit L Sq—C Rq [g] = (R Kq—S Kq [g]=) M S × S N—
k 2. ax. 1. M R × R N. & [k] proinde L Sq ┼ M R × R N = C Rq ┼ M S × S N. Verùm (ob ordinatim applicatas L S, C R) eſt L Sq . M S × S N [l] :: C Rq. M R × R N. ergo L Sq = M S × S N. & C Rq = M R × R N. (Nam ſi L Sq ⊏ vel ⊐ M S × S N; [m] eſſet hinc L Sq
l 21. 1. hujus.
m 25. 5.
n conv. 35. 3. ┼ M R × R N. ⊏ vel ⊐ C Rq ┼ M S × S N, contra modo oſtenſa). [n] Unde ſectio A B C circulus eſſet (non verò Ellipſis) contra hypotheſin.

Prop. XLIX. Probl. 6.

Fig. 149. Datâ coni ſectione, & puncto (A), non intra ſectionem, dato, ab eo ducere rectam lineam (A D), quæ ſectionem contingat.

* 47. hujus. 1. Caſ. Sectio data primò ſit Parabola, cujus * axis B E punctum datum A ſit in ipſa ſectione.

Analyſis. Tangat A D, axi B C occurrens in D, ductâque A E
a 35. hujus. ad B C perpendiculari, [a] erit B E = B D. Itaque

Compoſ. Si ex dato puncto A ducatur A E perpendicularis axi B C,
b 33. 1. hujus. & in axe producto ſumatur B D = B E, jungatúrque D A, liquet [b] hanc parabolen tangere.

2. Caſ. Punctum A ſit in ipſo axe.

Fig. 150. *Analyſis.* Tangat A D; ductáque ſit D E axi B C perpendicu-
c 35. 1. hujus. laris: itaque rurſus eſt A B [c] = B E.

Compoſ. Sumatur B E = A B; & ab E ducatur E D axi perpen-
d 33. 1. hujus. dicularis; ſectioni occurrens in D, & connectatur D A; [d] continget hæc ſectionem.

3. Caſ. Sin datum punctum coincidat vertici B, [c] liquet ductam per B axi
c 17. 1. huj. perpendicularem eſſe contingentem.

4. Caſ. Punctum A datum ſit alibi extra axem.

Fig. 151. *Analyſis.* Tangat A D. itaque ductâ A E ad axem B C parallelâ,
f cor. 46. & 35. 1. hujus. & ordinatim applicatâ D E, erit rurſus G E [f] = A G.

Compoſ. Per A ducatur A E axi parallela, factóque G E = A G,
g 46. & 33. 1. hujus. per E ordinatim applicetur E D, ſectioni occurrens in D, & conjungatur D A: [g] liquet hanc ſectionem contingere.

h 47. hujus. k 45. hujus. Sectio data ſit ſecundò Hyperbola; cujus [h] axis tranſverſus K B, [k] centrum F, aſymptoti F L, F M.

1. Caſ. Punctum datum A ſit in ſectione.

Fig. 152. *Anal.* Tangat A D; & ſit A E perpendicularis axi B C; eſt igi-
m 36. 1. hujus. tur K E. E B [m] :: K D. D B. *Com-*

Compos. E puncto A ducatur A E axi perpendicularis, [n] secetúrque K B in D, ut sit K D. D B :: K E. E B; jungatúrque D A. [o] Continget hæc sectionem. [n] 16. 6. [o] 34. 1. *hujus.*

Punctum A sit in axe. 2 *Cas.*

Anal. Tangat A D, & sit D E axi perpendicularis: itaque rursus K E. E B :: [m] K A. A B. Fig. 153.

Compos. * Fiat K E. E B :: K A. A B. & per E ducatur axi perpendicularis E D, sectioni occurrens in D; & connectatur D A; [o] hæc sectionem continget. * *Not.* 1.

Sit datum punctum A alicubi intra angulum L F M. 3 *Cas.*

Anal. Tangat A D, junctáque F A producatur, & fiat F O = F N; & *ordinatim applicetur D E. Est ergo O E. E N [p] :: O A. A N. Fig. 154. * [p] 36. 1. *hujus.*

Compos. Juncta F A producatur, & sumatur F O = F N. fiátque O E. E N [q] :: O A. A N. & ordinatim applicetur E D; jungaturq; D A. [r] Tanget hæc sectionem. [q] *Not.* 1. [r] 34. 1. *huj.*

Sit punctum A in F M unâ asymptoton. 4 *Cas.*

Anal. Tangat A D sectionem, asymptoto F L occurrens in P; sitque D Q ad L F parallela: atque ob A D [s] = D P, [t] erit A Q = Q F. Hinc Fig. 155. [s] 3. *hujus.* [t] 2. 6.

Compos. Bisecetur A F in Q, & per Q ducatur Q D ad F L parallela, sectioni occurrens in D, jungatúrque D A. Continget hæc sectionem. Nam productâ A D in p, ob A Q [v] = Q F, [x] erit A D = D P. [y] quare A D tangit sectionem. [u] *constr.* [x] 2. 6. [y] 9. *hujus.*

Punctum A sit in loco, qui deinceps est angulo L F M sectionem continenti. 5. *Cas.*

Anal. Tangat A D; junctáque A F producatur, cui parallela utcunque in sectione sumatur R S; & bisectâ R S in E, connectatur E F, & fiat F O = F N; est igitur E O diameter [z] ipsi A F conjugata; & per D ductâ D T ad E O parallelâ, erit [a] A F × F T quarta pars figuræ ad O N. Hinc Fig. 156. [z] 2. 37. *hujus.* [a] *cor.* 38. 1. *huj.*

Compositio. Juncta A F producatur, eique parallela utcunque ducatur R S (sectioni occurrens punctis R, & S); bisecetur R S in E; junctáque E F producatur, & fiat F O = F N, [z] ergo O N est transversa diameter, ipsi A F conjugata. * Fiat A F × F T æqualis quartæ parti figuræ ad O N, & per T ducatur T D ad O N parallela, sectioni occurrens in D, & jungatur D A. [b] Tanget hæc sectionem. * *Not.* 2. [b] *conv.* 38. 1. *huj.*

Sim-

6. Caſ. Sin aſſignetur punctum A intra angulum Y F Z, nulla inde tangens
c 31. 1. hujus. duci poterit; ducta enim linea c utramque Y F, Z F ſecabit: ergo non tanget.

Sit tertiò, ſectio data ellipſis, cujus axis B C, centrum F.

1 Caſ. Datum punctum A ſit in ſectione.

Anal. Tangat A D, & A E ordinatim applicetur ad axem. Eſtq;
d 36. 1. hujus. C E. E B :: d C D. D B.

Compoſ. Ducatur A E perpendicularis axi C B; & producatur
Fig. 157. C B, ut ſit C E. E B e :: C D. D B. & connectatur D A. f Tanget
e Not. 2. hæc Ellipſin.
f 34. 1. hujus.

2 Caſ. Punctum A ſit extra ſectionem.

Anal. Tangat A D; & juncta A F producatur, & ordinatim applicata ſit D E. Eſt itidem O A. A N g :: O E E N.
g 36. 1. hujus.

Compoſ. Connectatur A F ſectioni occurens in N, O; fiátque O A.
Fig. 158. A N h :: O E. E N. & per E ad A O ordinatim applicetur E D, ſecti-
h 10. 6. oni occurrens in D; & connectatur D A: k tanget hæc ſectionem.
k 34. 1. hujus

Fig. 159. *Not* 1. Datur recta K B ſecta in A; oportet producere hanc ad E, ita ut ſit K E. B E :: K A. A B.

a 12. 6. a Fiat K A — A B. A B :: K B. B E. ergo componendo erit K A. A B :: K E. B E. *Q. E. F.*

Prop. L. Probl. 7.

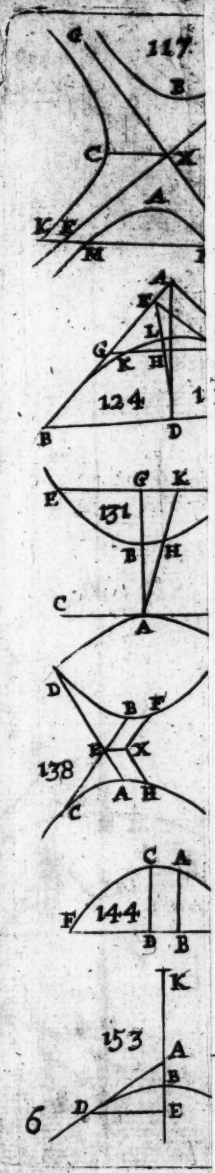
Fig. 160. Datâ coni ſectione, lineam contingentem ducere, quæ cum axe ad partes ſectionis angulum faciat, dato angulo acuto æqualem.

Sit ſectio primùm parabole, cujus axis A B.

Fig. 161. *Analyſis.* Tangat D C ſectionem, faciens angulum D parem dato
a 40. dat. F. Itaque ductâ C B ad A B perpendiculari, a datur ratio D B ad
b 33. 1 hujus. B C; ergo datur ratio A B b ($\frac{1}{2}$ D B) ad B C. c ergo datur angulus
c 41. dat. B A C. d quare datur poſitio rectæ A C, & inde punctum C, &
d 29. dat. hinc tangentis C D poſitio.

Compoſ. Sumpto E puncto utcunque in latere dati anguli ducatur E G ad alterum latus F G perpendicularis, & biſectâ F G in H, jungatur H E, fiátque ang. B A C = ang. G H E: & ab occurſu C ducatur C B ad A B perpendicularis; productâque A B, ſumatur A D
f 35. 1. hujus. g constr. et 15. = A B; & connectatur D C, f liquet hanc tangere ſectionem. Et
h 4. 6. (h quoniam F G. H G k :: D B. A B. & H G. G E k :: A B. B C (ob
k constr. ang. G H E k = B A C, & k rectos ad G & B) erit ex æquo, F G.

G E.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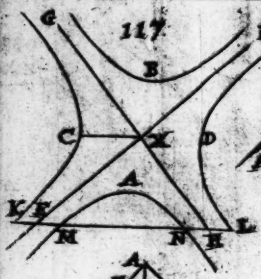
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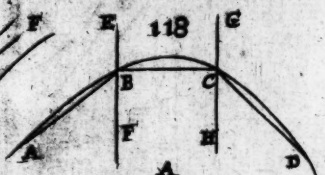
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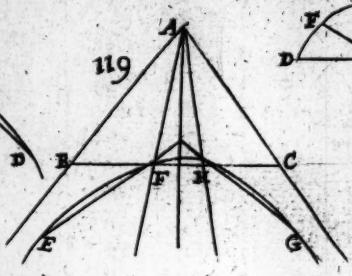
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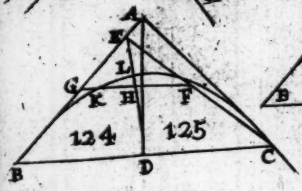
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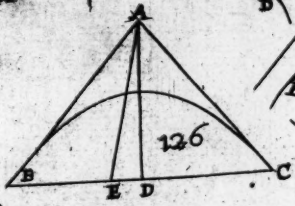
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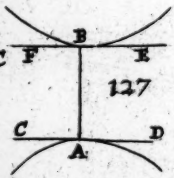
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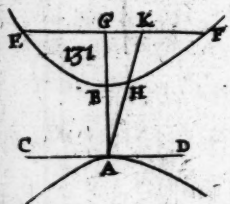
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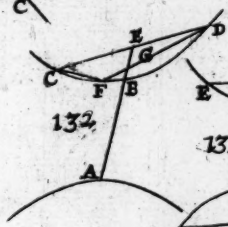
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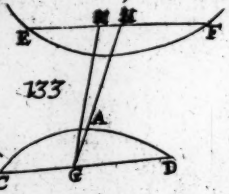
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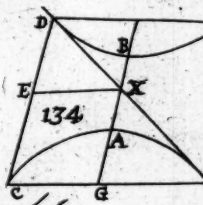
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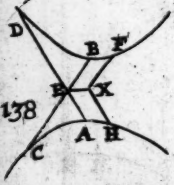
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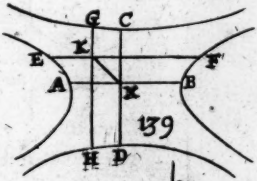
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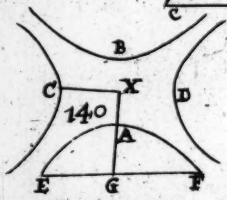
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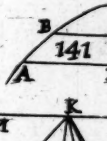
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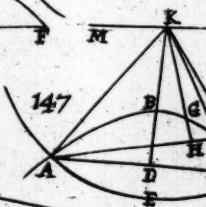
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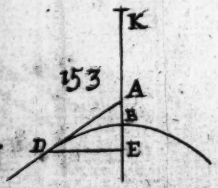
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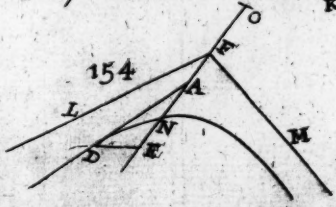
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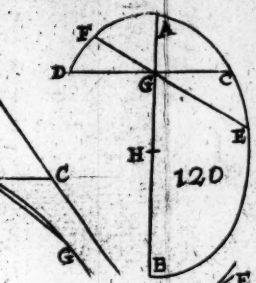

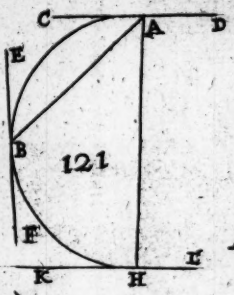

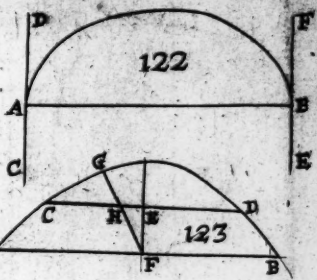

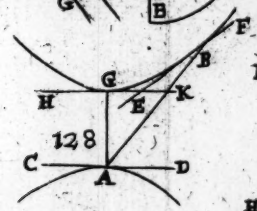

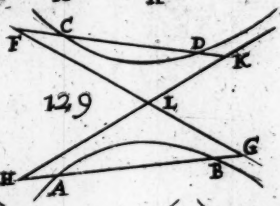

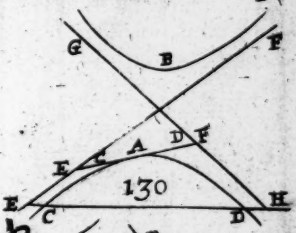

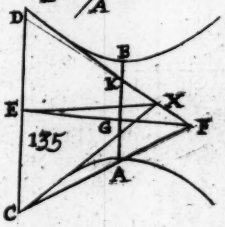

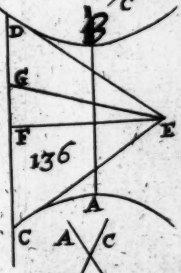

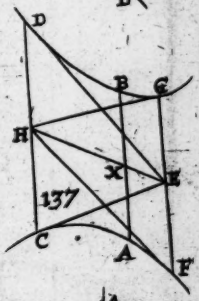

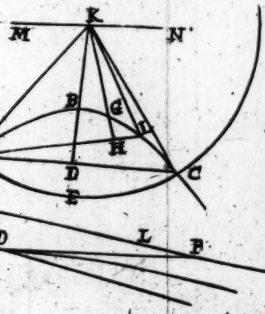

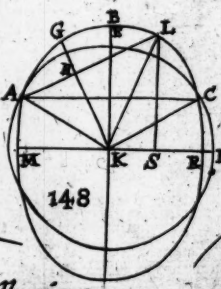

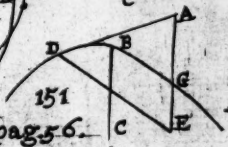

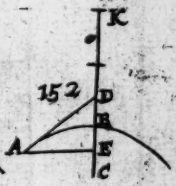

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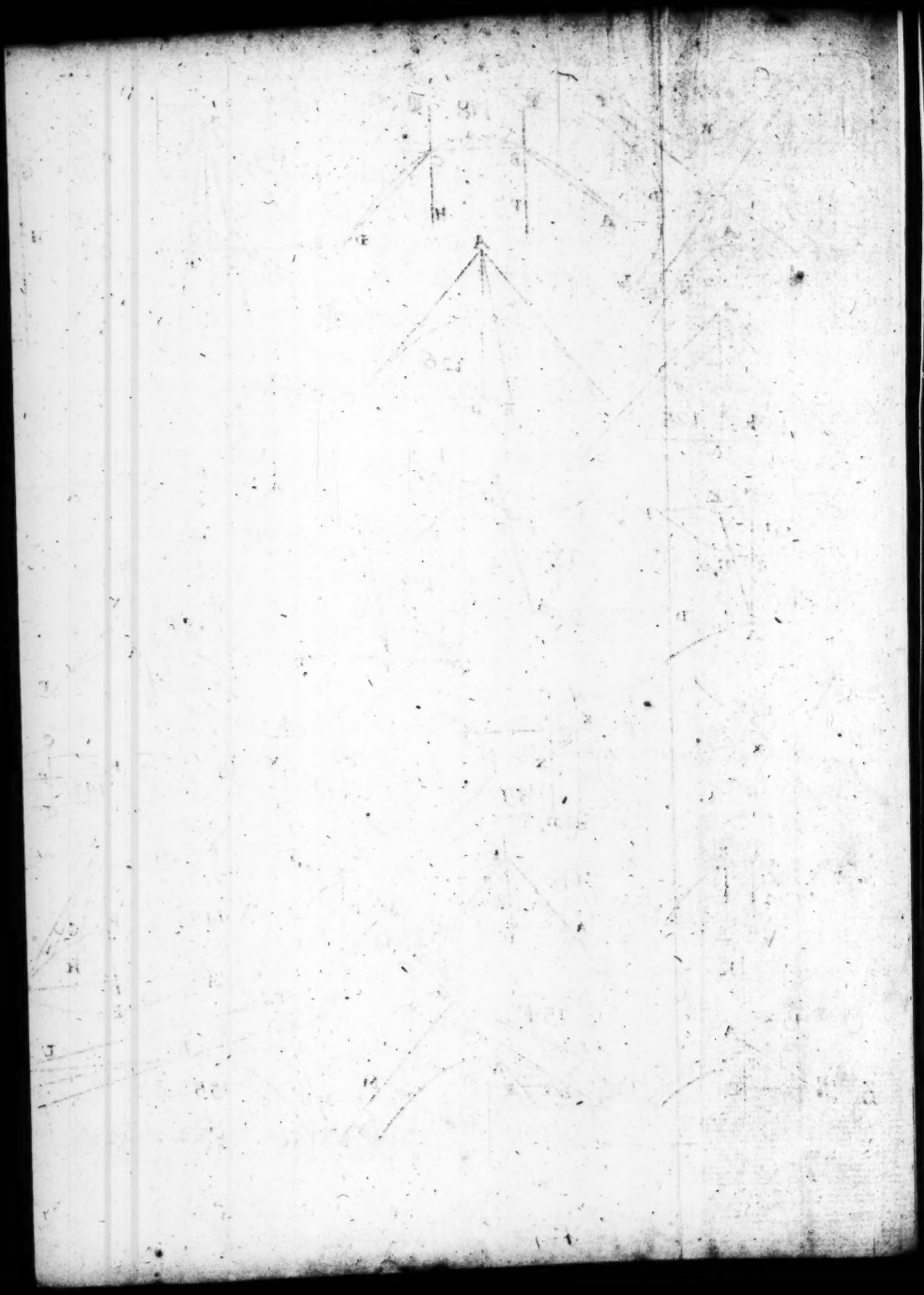

GE :: DB. BC. [l] ergo pares ſunt anguli F, D. *Q. E. F.* [l] 6. 6.

Fig. 162.

Sit ſecundò Hyperbole, cujus axis A B, centrum X.

Fig. 163.

Anal. Tangat D C, faciens angulum parem dato K H G; jungatúrque X C, & ſtatuatur C E axi perpendicularis; eſt igitur data ratio X E × E D ad E Cq [m] (eadem quæ T ad R); item [n] datur ratio E C ad E D (ob datos angulos E D C, & rectum E). [o] ergo datur ratio X E × E D ad E Dq; [p] hoc eſt ratio X E ad E D. Proinde [q] datur ratio X E ad E C. [r] ergo datur angulus E X C, & [s] hinc poſitio rectæ X C, & hinc punctum C, & hinc tangens C D.

[m] 37. 1. *hujus.* [n] 40. *dat.* [o] 50. & 8. *dat.* [p] 1. 6. [q] 8. *dat.* [r] 41. *dat.* [s] 29. *dat.*

Quòd ſi ducatur aſymptotos X F; [t] liquet angulum E D C majorem eſſe angulo A X F, quoniam D C [v] producta ipſam X F ſecabit. Itaque datus angulus non debet eſſe minor illo, quí ſectionem continet.

[t] 16. 1. [v] 3. *hujus.*

Compoſ. Sumatur G punctum utcunque in latere H G dati anguli; & à G ducatur ad H K perpendicularis G K. [y] Fiat autem T. R :: M K × K H. K Gq. & connectatur M G: dein fiat ang. A X C = [z] ang. K M G; & ab occurſu C [z] ducatur tangens C D. Dico factum.

[y] *Not.* [z] 49. *hujus.* [a] *conſtr. & 4. 6.*

Ducatur enim C E axi perpendicularis: & propter X E. E C [a] :: M K. K G. ac [b] ideò X Eq. E Cq :: M Kq. K Gq. & E Cq. X E × E D [c] :: (R. T :: [d]) K Gq. M K × K H; erit ex æquo X Eq. X E × E D. :: M Kq. M K × K H. [e] hoc eſt X E. E D :: M K. K H. Verùm E C. X E [f] :: K G. M K. Ergo rurſus ex æquo E C. E D :: K G. K H. ergo quum anguli E, K recti ſint, [g] erit ang. E D C = K H G. ergo factum.

[b] 22. 6. [c] 37. 1. *hujus.* [d] *conſtr.* [e] 1. 6. & 11. 5. [f] *priùs & inverſè.* [g] 6. 6.

Quòd verò X C ſectioni occurrit, ſic oſtenditur: Ducatur A F ad X A perpendicularis, angulóque A X F fiat æqualis K H L. Cùm igitur ſit M K × K H. K Gq (:: [h] T. R [k] ::) X Aq. A Fq [l] :: H Kq. K Lq) [m] ⊏ H Kq. K Gq. [n] erit M K × H K ⊏ H Kq & proinde M Kq ⊏ M K × H K. & M Kq. K Gq [o] ⊏ (M K × H K. K Gq [p] ::) X Aq. A Fq. quare X Eq. E Cq ⊏ X Aq. F Aq. & X E. E C [q] (hoc eſt X A. A N) ⊏ X A. A F. [r] ergo A N ⊐ A F. quare X C ſecat angulum E X F; & [s] propterea ſectioni occurrit.

[h] *conſtr.* [k] 1. *hujus.* [l] 4. & 22. 6. [m] *conſtr. et* 8. 5 [n] 10. 5. [o] 8. 5. [p] *priùs.* [q] 4. 6. [r] 10. 5. [s] 3. *hujus.*

Sit tertiò ſectio ellipſis, cujus axis A B, centrum X.

Fig. 164.

Analyſis. Tangat C D, faciens angulum D parem dato G; jungatúrque X C, & ſit C E axi perpendicularis. Eſt itaque data ratio C Eq ad E D × E X [a] (eadem quæ R ad T). Item ratio C Eq ad E Dq [b] datur (ob datos angulos D, & E); [c] ergo datur ratio E Dq ad E D × E X. [d] hoc eſt ratio E D ad E X. [e] ergo ratio E X ad C E datur. [e] quare datur angulus E X C, & hinc [f] poſitio rectæ X C, & inde punctum C, adeóq; [g] poſitio tangentis C D.

[a] 37. 1. *hujus.* [b] 40. *dat.* [c] 8. *dat.* [d] 1. 6. [e] 41. *dat.* [f] 29. *dat.* [g] 49. *hujus.*

Compoſ. Sumpto utcunque puncto F in latere dati anguli, ducatur F H ad G H perpendicularis; & fiat R. T :: [h] F Hq. G H × H K; junctâque K F, fiat ang. A X C = H K F; & ab occurſu C [k] ducatur tangens C D. Dico factum. Nam ductâ C E ad A B perpendicularí, propter X Eq. E Cq [l] :: K Hq. H Fq. & E Cq. E D × X E :: [m] R. T :: [n] H Fq. H G × K H. erit ex æquo X Eq. E D × X E :: K Hq. H G × K H. [o] hoc eſt X E. E D :: K H. H G. Item E C. X E :: [p] H F. H K. ergo rurſus ex æquo E D. E C :: H G. H F. Unde cùm ang. E [n] = H, [q] erit quoque ang. D = G. Ergo factum.

h Not.
k 49. hujus.
l conſtr. & 4. & 22. 6.
m 37. 1. hujus.
n conſtr.
o 1. 6.
p conſtr. et 4. 6.
q 6. 6.

Not. Fieri debet R. T :: F Hq. G H × H K. Itaque $\frac{T \times FHq}{R \times GH} = HK$.

Compoſ. Fiat R. T :: F H. Q = $\frac{T \times FH}{R}$. tum G H. Q :: F H. $HK = \frac{Q \times FH}{GH} = \frac{T \times FH \times FH}{R \times GH}$

Prop. LI. Probl. 8.

Fig. 166.

Datâ coni ſectione lineam contingentem ducere, quæ cum diametro per tactum ducta faciat angulum dato angulo acuto (Z) æqualem.

a 50. hujus.
b cor. 46. 1. h.
c conſtr.
d 29. 1.

In parabola facilè conficitur. [a] Tangat enim hanc utcunque C D faciens cum axe A B angulum D parem dato Z; & per tactum C ducatur C E ad A B parallela: [b] Liquet C E eſſe diametrum, & angulum D C E alterno C D A, [c] (hoc eſt dato Z) [d] æquari. *Q. E. F.*

Fig. 167. 168.

In hyperbola verò, cujus axis A B, centrum E.

Analyſis. Tangat C D faciens angulum E C D parem dato; Trigono autem C D E circumſcribatur circulus; & ductâ per C ad axem perpendiculari G C Z, per V centrum circuli ducantur V Q ad Z G, & V Y ad E G parallelæ, & ſit C S ad G E parallela. Eſtque Z G. C G [e] :: (Z G × G C. C Gq [f] :: E G × G D. C Gq [g] ::) T. R. ergo dividendo M C. C G :: T—R. R. & bipartiendo antecedentes [h] Y C. C G. [k] (hoc eſt V S. S Q) :: $\frac{T-R}{2}$. R. Hinc

e 1. 6.
f 36. 3. & 7. 5.
g 37. 1. hujus.
h 3. 3.
k 34. 1.

Compoſ. Exponatur utcunque recta F H, ſuper quâ [l] deſcribatur ſegmentum circuli capiens angulum (ut F K H) parem dato Z. A circuli autem centro N ducatur N O ad F H perpendicularis, [m] ſecetúrque

l 33. 3.
m 12. 6.

túrque N O in P, ut ſit N P. P O :: $\frac{T-R}{2}$. R. ducatúrque P K ad F H parallela, circulúmque ſecans in K, & per K ducatur K L ad protractam F H perpendicularis; & producatur L K M, & perpendicularis huic ducatur N X. Junctâ denique K F, fiat ang. A E C = ang. L F K; & per occurſum C [n] ducatur ſectionem contingens C D. Dico factum.

[n] 49 *hujus.*

Ducatur axi perpendicularis C G; & quoniam $\frac{T-R}{2}$. R. [o] :: N P. P O (X K. K L) erit (duplando antecedentes) T — R. R :: M K. K L. & componendo T. R :: M L. K L :: [p] M L × K L. [q] (F L × H L). K Lq :: T. R :: [r] E G × D G. C Gq. Eſt ergo F L. K L + H L. K L [s] (F L × H L. K Lq) = E G. C G + D G. C G (E G × D G. C Gq). atqui (ob [t] ſimilia trigona F L K, E G C) [v] eſt F L. K L = E G. C G. ergo H L. K L :: D G. C G. [x] ergo anguli H K L, D C G pares ſunt, & [y] proinde reliqui F K H, E C D etiam pares ſunt. *Q. E. D.*

[o] *conſtr.* [p] 1. 6. [q] 36. 3. [r] 37. 1. *hujus.* [s] 23. 6. [t] *conſtr.* [u] 4. 6. [x] 6. 6. [y] 3. *ax.* 1. [z] 1. *hujus.*

Quòd verò E C ſectioni occurrat, ſic oſtenditur; ſit E T aſymptotos, & axi A B ducatur perpendicularis A T. Eſtque E A q. A T q [z] :: (T. R :: [a] F L × L H. L Kq) [b] ⊐ F Lq. K Lq, [c] vel E Gq. C Gq. ergo ang. A E T ⊏ ang. A E C; [d] & proinde E C ſectionem ſecat.

[a] *conſtr.* [b] 8. 5. [c] *prius.* [d] 2. *hujus.*

Coroll. Si ſit F L × H L. K Lq :: E G × D G. C Gq. erunt trigona H L K, D G C ſimilia.

Prop. LII.

Si ellipſim (cujus Axes A B, C D, & Centrum E) recta linea (G L) contingat; angulus (L F E), quem facit cum diametro (E F) per tactum (F) ducta, non eſt minor angulo (L C A) deinceps ei (A C B), qui lineis (A C, B C) ad mediam ſectionem (C) inclinatis continetur. Fig. 169. 170. 171.

Sit primò E F ad B C parallela: ergo cùm ſit A E [a] = E B, [b] erit A H = H C ergo, cùm E F [c] ſit diameter, [d] erit A C tangenti G L parallela: unde ang. L F H [e] = (C H E [e] =) L C H.

[a] *hyp.* [b] 2. 6. [c] *cor.* 47. 1. *huj.* [d] 5. *hujus.*

Sed non ſit E F ad B C parallela. ergo ductâ F K ad A B perpendiculari [f] erunt anguli L B E, F E K inæquales, & trigona C B E, F E K diſſimilia. Non igitur eſt E Kq. K Fq :: (B E q. (A E × E B). E Cq; [g] hoc eſt T. R; [g] hoc eſt) G K × K E. K Fq. ergo E Kq, & G K × K E ſunt inæqualia, & proinde G K, K E inæquales ſunt. [h] Capiat circuli ſegmentum M Y N angulum parem angulo A C B: [k] ergo id ſemicirculo majus eſt (nam ob A E, vel B E [l] ⊐ E C, [m] eſt uterque angulus A C E,

[e] 29. 1. [f] *cor. ult. def.* [g] *ad* 16. 1. *huj.* [h] 33. 3. [k] 31. 3. [l] *hyp.* [m] 18. 1. & 32.

n 10. 6. A C E, B C E ſemirecto major, ideóque ang. A C B obtuſus). n Fiat N X. X M :: G K. K E & per X ducatur ipſi M N perpendicularis Y X, & connectantur M Y, N Y; biſectâque M N in T, erigatur ei perpendicularis O T P, in qua ſumpto circuli centro R, ducatur R S ad Y X perpendicularis; jungantúrque M O, N O: liquet ang. N O T o (½ N O M) angulo B C E p (½ B C A) æquari; ideóque eſſe T Nq. T Oq q :: E Bq. E Cq. Eſt verò Y S. T R q (X S) r ⊐ O R. T R. & conversè S Y. X Y ⊏ R O. T O. & duplando antecedentes Z Y. X Y. ⊏ P O. T O. & dividendo Z X. X Y. (o hoc eſt Z X × X Y p (vel N X × X M). X Y q) ⊏ (P T. T O q :: T Nq. T Oq r :: E Bq. E Cq r ::) G K × K E. K Fq. Itaque ſi fiat N X × X M. X Vq :: G K × K E. K Fq, s erit X V ⊏ X Y. Et quoniam N Xq. N X × X M t :: (N X. X M v :: G K. K E t ::) G Kq. G K × K E. erit ex æquo N Xq. X Vq :: G Kq. K Fq. x & N X. X V :: G K. K F; connexâ igitur N V, y erit ang. N V X = ang. G F K. & ſimili diſcurſu ang. M V X = E F K z unde totus ang. N V M = G F E. atqui ang. N V M a ⊐ N Y M (A C B). ergo ang. G F E ⊐ A C B, & qui deinceps ang. E F L ⊏ L C A. Q. E. D.

o 4. 1.
p 4. & 22. 6.
q 34. 1.
r 8. 5.
o 1. 6.
p 35. 3. (6.
q cor. 13. & 20.
r prius.
s. 10. 5.
t 1. 6.
u conſtr.
x 22. 6.
y 6. 6.
z 2. ax. 1.
a 21. 1.

Prop. LIII. Probl. 9.

Fig. 172. 173. 174. 175.

Datâ ellipſi (A B C D) contingentem lineam ducere, quæ cum diametro per tactum ducta faciat angulum dato angulo acuto (Y) æqualem. * Oportet autem acutum angulum datum non eſſe minorem angulo (A C G) deinceps ei (A C B), qui lineis (A C, B C) ad mediam ſectionem inclinatis continetur.

* juxta præcedentem.

Sit primò datus ang. Y = A C G per centrum E ducatur E K ad B C parallela; & per occurſum K ſectionem contingens G H. Dico factum.

Nam ob A E a = E B, b erit A F = F C: ergo cùm K E ſit diameter, c erit A C tangenti H G parallela. ergo ang. E K G d = (E F C d = A C G = e) Y. Q. E. F.

a. byp.
b. 2. 6.
c. 5. hujus.
d. 29. 1.
e. byp.

Sin. ang. Y ⊏ A C G, erit qui deinceps Z ⊐ A C B. Exponatur utcunque circulus, f in quo ſegmentum M N P capiat angulum parem angulo Z; biſectâque M P in O, ducatur per O ipſi M P perpendicularis N R g (in quo circuli centrum V), & connectantur M N, P N. Itaque ang. A C E h (½ A C B) ⊏ ang. M N O h (½ M N P, i vel ½ Z). quare A E. E C ⊏ M O. O N. & A Eq. E Cq. p (T. R) ⊏ (M Oq. q (N O × O R). O Nq r ::) O R. O N. ergo componendo T + R. R ⊏ R N. O N. & bipartiendo

f. 33. 3.
g cor. 1. 3.
h. 4. 1.
k. conſtr.
l cor. ad defin. ad 16. 1. huj.
q 35. 3.
r 1. 6.

endo antecedentes $\frac{T+R}{2}$. R ⊏ V N. O N. dividendóque $\frac{T-R}{2}$. R ⊏ V O. O N. Sit $\frac{T-R}{2}$. R :: V O. O I [s] (⊐ O N). & per I ducatur I X ad M P parallela, & per X ipſa X S T ad N R parallela, & V Q ad M P parallela. ergo cùm $\frac{T-R}{2}$. R [t] :: (V O. O I [u] ::) Q S. S X. erit componendo $\frac{T+R}{2}$. R :: Q X, S X. & duplando antecedentes, T+R. R :: T X. S X. & dividendo T. R :: T S. S X. Connectantur igitur M X. P X, & fiat ang. A E K = ang. M P X. & per occurſum K ducatur G H tangens ſectionem. Dico factum. Nam ordinatim applicetur K L. éſtque H L × L E. K Lq [x] :: (T. R [y] :: T S. S X [z] ::) T S × S X [a] (M S × S P). S X q. [b] ergo H L. K L + L E. K L [b] (H L × L E. K Lq = M S × S P. S X q [c] =) M S. S X + S P. S X. atqui ([d] ob ang. A E K = M P X, & rectos ad L, & S) [e] eſt L E. K L :: S P. X S. ergo manet H L. K L :: M S. X S. ergo ang. H K L = M X S. ergo totus ang. H K E [f] = (ang. M X P [g] =) ang. Z. & qui deinceps ang. E K G = ang. Y. Ergo factum.

s 10. 5.
t *conſtr.* (5.
u 34. 1. & 11.
x 21. 3.
y *prius.*
z 1. 6.
a 35. 3.
b 23. 3.
c 23. 6.
d *conſtr.*
e 4. 6.
f 2. *ax.* 1.
g *conſtr.*

Coroll. Si H L × L E. K Lq :: M S × S P. X S q. Erit trigonum H L K ſimile trigono M S X.

Problema.

Linea E F coni ſectionem ſecet, oportet huic parallelam ducere, quæ ſectionem contingat. Biſecetur E F in C; & per C ducatur diameter ſectioni occurrens in T; & per T ducatur T S ad E F parallela, hæc ſectionem continget. Res liquidò patet. Fig. 176.

APOL-

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CONICORUM
LIB. III.

Prop. I.

Fig. 177. 178. 179. 180. SI coni ſectionem, vel circuli circumferentiam contingentes rectæ lineæ (A C, B D) inter ſe conveniant (in E), pérque tactus (A, B) ducantur diametri (A D, B C), quæ contingentibus occurrant (in D, C); triangula (A E D, B E C) ad verticem facta, ſibi ipſis æqualia erunt.

Per A ducatur A F ad B D parallela. Eſtque (in parabola) *p g r.* A D B F [a] = triang. A C F: ablatóque communi trapezio A E B F, reſtat triang. A E D = triang. B E C. *Q. E. D.*

In aliis verò ſectionibus (ob G F. G B [b] :: G B. G C) [d] eſt G F. G C. (hoc eſt triang. G F A. G C A) [d] :: (G Fq. G Bq [e] ::) triang. G F A. G B D. [f] quare triang. G C A = G B D. & proinde (ablato vel addito communi quadrilatero) remanet triang. A E D = triang. B E C. *Q. E. D. Coroll.* Triang. G C A = triang. G B D.

a 42. 1. *hujus.*
b 37. 1. *hujus.*
c 1. 6.
d *cor.* 20. 6.
e 1, & 22. 6.
f 9. 5.

Prop. II.

Fig. 181. 182. 183. 184. 185. 186. Iiſdem poſitis, ſi in coni ſectione, vel circuli circumferentia, ſumatur aliquod punctum (G), & per ipſum ducantur (K G L, F G M) æquidiſtantes contingentibus uſque ad diametros; quadrilaterum (G L G I) factum ad (A C) unam contingentium, & ad (B C) unam diametrorum, æquale erit triangulo (A I M), quod ad eandem contingentem (A C) & ad alteram diametrum (A D) conſtituitur.

Nam

Nam ob triangulum GKM [a] æquale quadrilatero AKLC, liquet (addito vel ablato communi AIGK) tota, vel reſidua AIM, GICL æquari. [a] 42, vel 43. 1. *hujus.*

Prop. III.

Iiſdem poſitis, ſi in coni ſectione, vel circuli circumferentia, ſumantur duo puncta (F, G) & per ipſa ducantur (FHKL, NFIM, & GHPR, NGXO) æquidiſtantes contingentibus, uſque ad diametros, quadrilatera ((LGRF, & LNRN) quæ ab ipſis fiunt, in diametris conſtituta, inter ſe æqualia erunt. Fig. 187. 188. 189.

Nam quadrilat. GOLH + 4lat. HLCP [a] = 4lat. GOCP [b] = triang. ARP = [a] 4lat. MIPR + triang. AIM [c] (hoc eſt = 4lat. MIPR + 4lat. FLCI [d]) = 4lat. MFHR + 4lat. HLCP. [e] ergo 4lat. GOLH = 4lat. MFHR. & [f] proinde etiam quadrilat. NOLF = quadrilat. NMRG. *Q. E. D.*

a 19. *ax.* 1.
b 2, 3. *hujus.*
c 2. *ax.* 1. &
2. 3. *hujus.*
d 2, & 3. *ax.* 1.
e 3. *ax.* 1.
f 2. *ax.* 1.

Prop. IV.

Si quæ oppoſitas ſectiones contingunt duæ rectæ lineæ (AC, BC) inter ſe conveniant (in C), & per tactus (A, B) ducantur diametri (ADH, BDG) contingentibus occurrentes (in F, G); triangula (ACF, BCG), quæ ad contingentes conſtituuntur, ſibi ipſis æqualia erunt. Fig. 190.

Per H ducatur tangens HL. [a] Hæc tangenti AG parallela erit; [b] & AD = HD. quare triang. AGD. [c] = (triang. HLD [d] =) triang. BFD. Proinde (addito communi GDFC) erit triang. ACF = triang. BCG. *Q. E. D.*

a *cor.* 44. *hujus.*
b 30. 1. *hujus.*
c 4. 6. & 2. 1.
d 1. 3. *hujus.*

Prop. V.

Si, quæ oppoſitas ſectiones contingunt, duæ rectæ (ED, FD) ſibi ipſis occurrant (in D), & in ſectionum quavis (B) ſumatur aliquod punctum (G), à quo ducantur duæ lineæ, una quidem GM æquidiſtans contingenti (FD), altera verò (GKHL) æquidiſtans ei (FE), quæ tactus (E, F) conjungit, triangulum (GHM), quod ab ipſis conſtituitur ad diametrum (CD) per occurſum ductæ, à triangulo (KHD), quod eſt ad occurſum contingentium, differt triangulo (FKL) facto ad contingentes, & ad diametrum (FC), quæ per tactum (F) ducta fuerit. Fig. 191.

Nam

a 29. & 38. 2. b. b hyp. c 45. 1. hujus.

Nam ob [a]diametrum CD, & ordinatim applicatam FE, [b] atque GL, GM ipſis FE, FD parallelas, [c] liquet eſſe triang. MGH = triang. CHL + CDF = triang. KHD + FKL.

Coroll. Triang. KFL = quadrilat. MGKD.

Prop. VI.

Fig. 192. 193.

Iiſdem poſitis, ſi in una oppoſitarum ſectionum ſumatur aliquod punctum (K), & ab eo ducantur rectæ lineæ (KLM, KNX) contingentibus (AF, BG) æquidiſtantes, quæ & contingentibus (AF, BG), & diametris (AEC, BED) occurrant; quadrilaterum (KF) ab ipſis factum ad unam contingentium (AF), & ad unam diametrorum (BD) æquale erit triangulo (AIN) quod ad eandem contingentem, & ad alteram diametrum (AC) conſtituitur.

a 2. 3. hujus. b 2. 3. hujus. c 4. 3. hujus. d 1. 3. hujus. e 2. ax. 1.

Nam (in 1. fig.) triang. KON [a] = 4lat. AOMF. unde (addito communi AIKO) triang. AIN = 4lat. KF. Item (in 2 fig.) ductâ contingente COP; [b] erit triang. CON = 4lat. KP. additóque communi OE, triang. CPE ([c] hoc eſt triang. BGE, [d] hoc eſt triang. AFE) [e] = 4lat. KE. quare itidem addito communi EI, [e] erit triang. AIN = 4lat. KF. *Q. E. D.*

Cor. Triang. AFE = 4lat. KE.

Prop. VII.

Fig. 194.

Iiſdem poſitis, ſi in utraque ſectione (AB, CD) ſumantur aliqua puncta (K, L), & ab ipſis ducantur contingentibus æquidiſtantes (MKPRX, NSTLW), quæ & contingentibus & diametris occurrant, quadrilatera (KT, L[illegible] & KY, LR) à lineis ductis conſtituta ad diametros, inter ſe æqualia erunt.

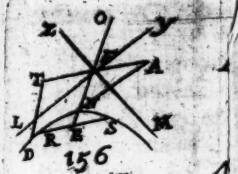
a 2. hujus. b 2. ax. 1. c cor. 6. hujus.

Nam ob quadrilat. KRFO [a] = triang. AOI, erit 4lat. KREI [b] = (triang. AEF [c] =) 4lat. LE. [b] unde 4lat. KRTN = 4lat. LI; [b] & 4lat. KXY[illegible] = LR. Quæ *E. D.*

Prop. VIII.

Fig. 195.

Iiſdem poſitis, pro punctis K, L ſumantur C, D, in quibus diametri (AC, BD) cum ſectionibus conveniant, & per ipſa ducantur contingentibus æquidiſtantes (DX, CT); dico quadrilaterum DG quadrilatero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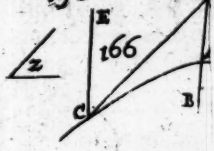
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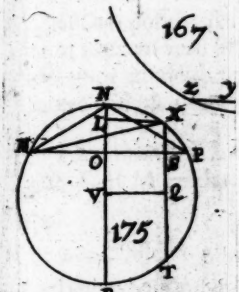
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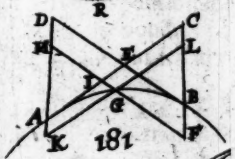
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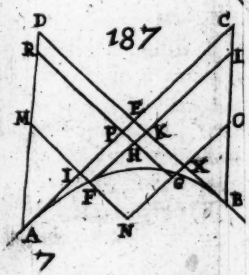
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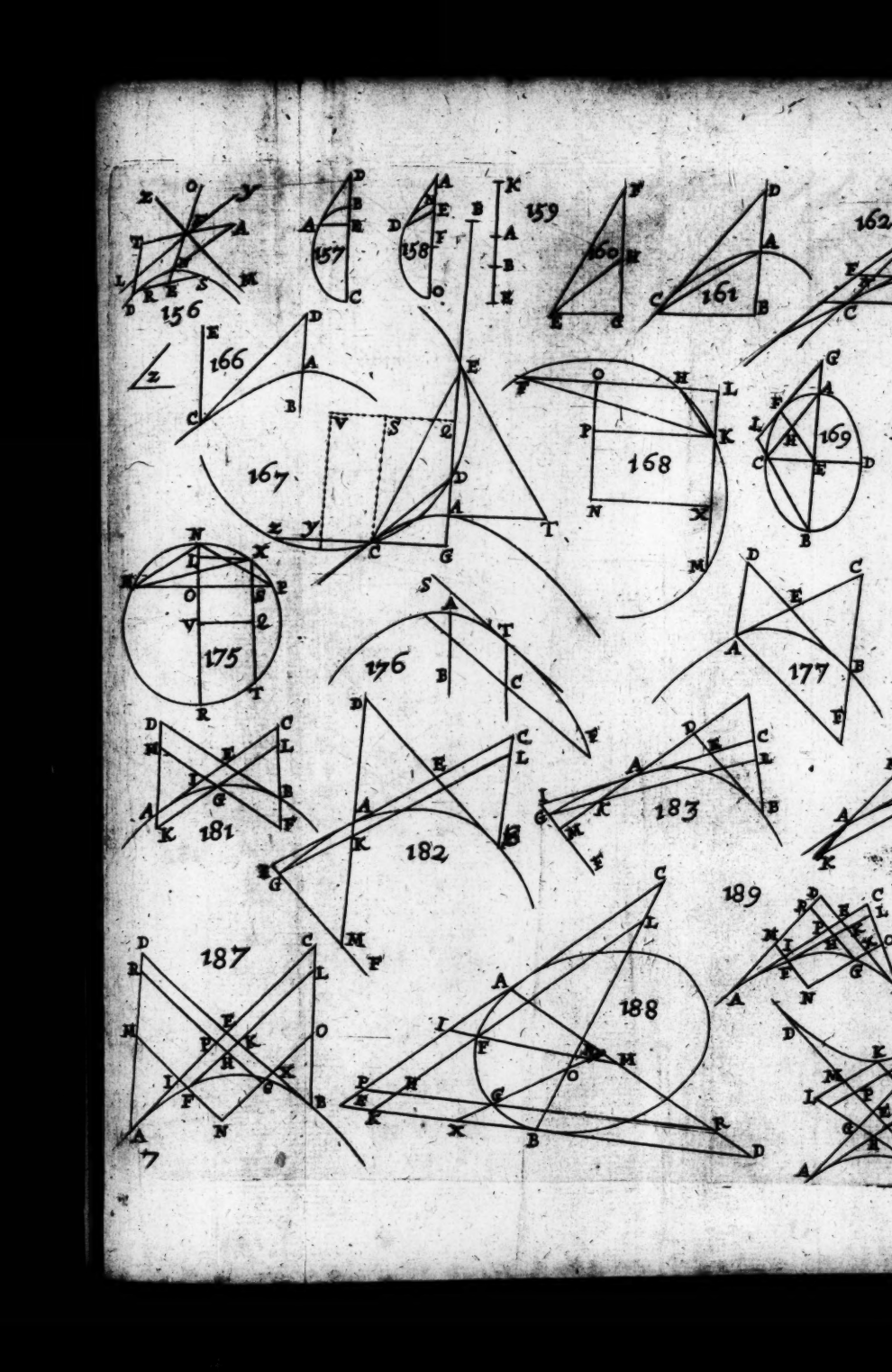
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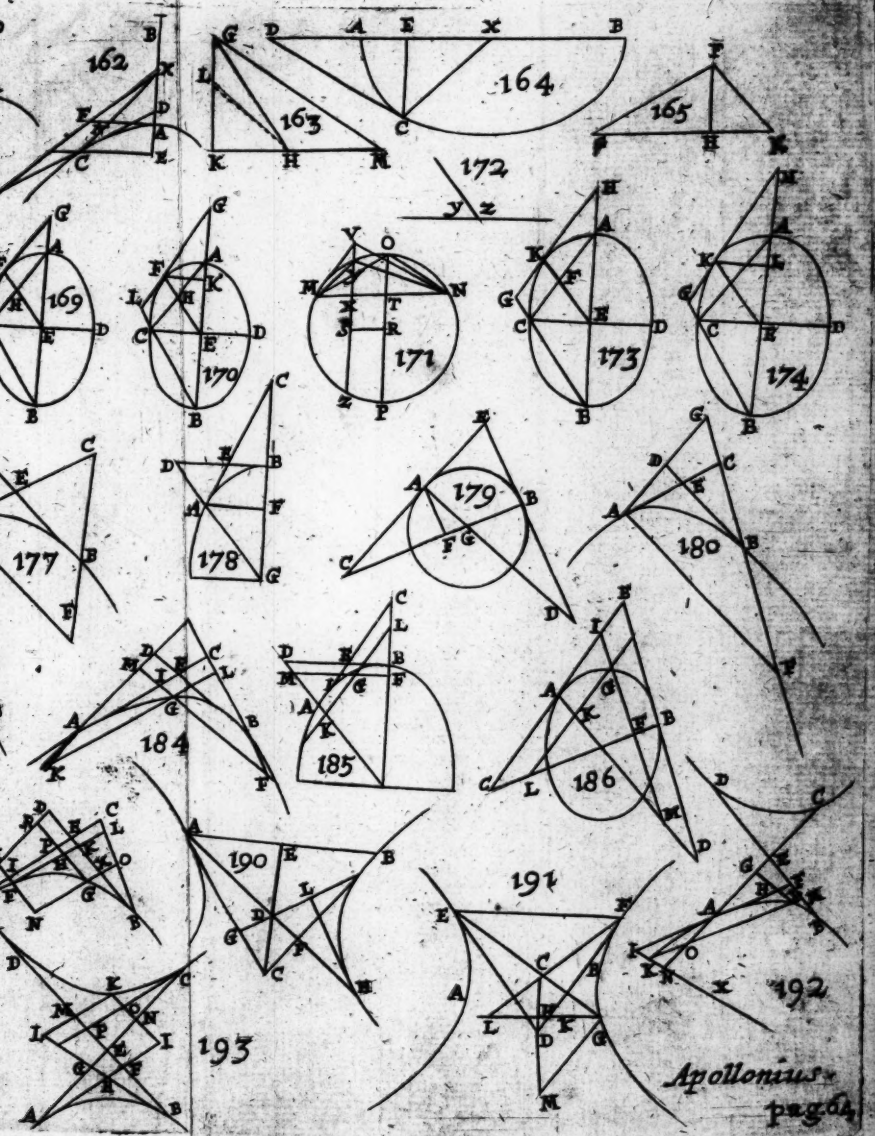
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drilatero F C, & quadrilaterum X I quadrilatero T O æquale eſſe.

Nam ob trigona A E F, B E G [a] æqualia, [b] erit A E. E G :: B E. E F; & [c] converſè, A E. A G :: B E. B F. item A C. A E :: ([d] 2. 1[d] ::) B D. B E. ergo ex æquali A C. A G :: B D. B F. ergo [e] triang. A C T. A G H :: triang. B D X. B F H. atqui triang. A G H [f] = B F H. ergo triang. A C T [g] = B D X: [h] quare 4lat. C H = 4lat. D H. & 4lat. C F = 4lat. D G. Adhæc triang. C E O [k] = triang A E [l] = triang. B E G [k] = triang. D E I. [m] ergo 4lat X T = T O. Quæ E. D.

a 1. *hujus.*
b 15. 6.
c *cor.* 19. 5.
d 30. 1.
e 22. 6.
f 1. *hujus.*
g 14. 5.
h 3. *ax.* 1.
k 8. 1.
l *cor.* 1. *hujus.*
m 2. *ax.* 1.

Prop. IX.

Iiſdem poſitis, ſi alterum quidem punctum, ut K, ſit inter diametros, alterum verò ſit idem quod unum punctorum C, D, ut C; & ducantur æquidiſtantes; dico triangulum C E O æquale eſſe quadrilatero K E, & quadrilaterum L O ipſi L M æquale eſſe. Fig. 196.

Nam triang. C E O [a] = (triang. A E F [b] =) 4lat K E. [c] ergo triang C R M = 4lat. K O. & 4lat. L M = 4lat. L O.

a 4. *hujus.*
b *cor.* 6. *huj.*
c 3. *ax.* 1.

Prop. X.

Iiſdem poſitis ſumantur K, L, non tamen in punctis, in quibus diametri ſectionibus occurrunt, demonſtrandum eſt quadrilaterum L T R X quadrilatero ω X K I æquale eſſe. Fig. 197.

Nam triang. T Y E — Y ω L [a] = (triang. B E G [b] = triang. A E F [c] =) triang X E I — X R K. [c] ergo triang. T Y E + X R K = triang. X E I + Y ω L. additóque utrinque ſpatio K X E Y L χ, [c] erit 4lat. L T R χ = ω χ K I. *Q. E. D.*

a 43. 1. *hujus.*
b 1. *hujus.*
c 1. *ax.* 1.

Prop. XI.

Iiſdem poſitis, ſi in quavis ſectione (A B) ſumatur punctum B, & ab ipſo lineæ æquidiſtantes ducantur; una quidem (B M) contingenti (A E) æquidiſtans, altera verò (B L) æquidiſtans ei (A D) quæ tactus conjungit; triangulum (B F M) quod ab ipſis fit ad diametrum (G M) per occurſum (E) contingentium ductam, à triangulo (A K L) contento lineâ contingente (A K) & diametro (A H L) per tactum, differt triangulo (K E F), quod ad contingentium occurſum conſtituitur. Fig. 198.

a 45. 1 *hujus.* b 19. *ax.* 1. c 3. *ax.* 1.

Nam triang. BMF[a] = (triang. LHF + triang. HAE[b] =) triang. AKL + triang. KFE.

Cor. 4 lat. BKEM[c] = triang. AKL.

Prop. XII.

Fig. 199.

Iisdem positis, si in una sectione (AB) sumantur duo puncta (B, K), & ab utrisque similiter ducantur æquidistantes (BLMN, KOXP ad AD; & BXRKLS ad AE); quadrilatera (BP, KR) ab ipsis constituta, æqualia erunt.

a *cor.* 11. *huj.* b 3. *ax.* 1. c 2. *ax.* 1.

Nam quia triang. AOP[a] = 4lat KOES; & triang. AMN[a] = 4lat BMER; [b] erit 4 lat. MOPN (= triang AOP — triang. AMN)[b] = (4lat KOES — 4lat BMER[b] =) 4lat KXRS — 4lat BMOX, [c] unde 4lat KXRS = (4lat MOPN + 4lat BMOX =) 4lat BXPN. *Q. E. D.*

Prop. XIII.

Fig. 200.

Si in oppositis sectionibus (A, B, C, D), quæ conjugatæ appellantur, rectæ lineæ (AE, BE) contingentes sectiones (A, B), quæ deinceps sunt, in unum punctum (E) conveniant, & per tactus (A, B) ducantur diametri (AHC, BHD); triangulâ (BFH, AGH), quorum communis vertex est sectionum centrum (H), inter se æquales erunt.

a 20. 2. *hujus.* b 4. 6. c 38. 1. *hujus.* d *Not.* e 11. 6. f 11. 5. g 9. 5. h 1. 6. k 15. 6.

Per puncta A, & H ducantur AK, LHM ad BE parallelæ; [a] liquetque esse LM, DB diametros conjugatas. Itaque KH. HB[b] (hoc est AH. HF)[c] : : HB. HG[d] ergo triang. AGH[e] = triang. BHF. *Q. E. D.*

Not. [a] Sit HX. HF : : (HB. HG[f] : : AH. HF)[g] ergo HX = AH. & ideo triang. AGH[h] = triang. HGX[k] = triang. BHF.

Prop. XIV.

Fig. 201.

Iisdem positis, si in quavis sectione (B) sumatur punctum (X), & ab ipso ducantur lineæ (XRS, XOT) æquidistantes contingentibus usque ad diametros (BHD, AHC); triangulum (OHT), quod ad centrum (H) constituitur, à triangulo (XTS) circa eundem angulum (T) differe triangulo (HBF, vel AGH) basin habenti lineam contingentem (BF, vel AG) & verticem sectionum centrum (H).

ducatur

Ducatur A Y ad B F parallela; & propter [a] conjugatas diametros L M, D B; & huic ordinatim applicatam A Y, [b] erit A Y. Y G. [c] (hoc eſt X T. T S) = H Y. Y A [c] (H B. B F) + T R. [d] quare triang. O H T = (triang. X T S + triang. B F H. [e] =) triang. X T S + triang. A G H. *Q. E. D.*

a 20. 2 *hujus.*
b 40. 1 *hujus.*
c 4. 6.
d 41. 1. *hujus.*
e 13. *hujus.* & 1. *ax.*

Prop. XV.

Si oppoſitarum ſectionum (A B, G S, T, X) quæ conjugatæ appellantur, unam (A B) contingentes rectæ lineæ (A D E, B D C) conveniant (in D), & per tactus (A, B) ducantur diametri (A H F, B H T); ſumatur autem punctum (S) in quavis (G S) ſectionum conjugatarum, & ab ipſo ducantur contingentibus æquidiſtantes (S L, S Y) uſque ad diametros (B T, A F) triangulum (S L Y), quod ab ipſis ad ſectionem conſtituitur, majus eſt quam triangulum (H L F), quod ad centrum (H), triangulo (H C B) baſin habenti lineam contingentem (B C) & verticem (H) centrum ſectionum. Fig. 202. 203.

Ducantur X H G ad B C, & G I K ad A E, & S O ad B T parallelæ. Fiántque D B. B E [a] :: M N. 2 B C [b] :: M P (½ M N). B C. atque X G. T B :: T B. R. Liquet X G × B T fore conjugatas diametros; & S O (vel L H) [c] ordinatim applicari ad X G. [d] & M N fore rectum latus figuræ ad B T, [e] & R rectum latus figuræ ad X G. Porro D Bq. D B × B E [f] :: (D B. B E [g] :: M P. B C [h] ::) M P × B H. [i] (H Gq). B C × B H. permutandóque D Bq. H Gq [m] (hoc eſt triang. D B E. triang. G H I) :: D B × B E. B C × B H [n] :: triang. D B E. triang. C B H. unde triang. G H I [o] = triang. C B H. Atqui H L. L F. [p] :: (H B. B C [q] :: H B. M P [r] (R. X G) + M P. B C [s] (D B. B E vel G H. H I) =) R. X G + G H. H I [t] = H L. L F. [v] unde triang. S L Y = (triang. H L F + triang. H G I [x] =) triang. H L F + triang. C B H. *Q. E. D.*

a 1 2. 6.
b 15. 5.
c 11. 6.
d 20. 2. *hujus.*
e 47. 1. *hujus.*
f 51. 1. *hujus.*
g 4. *def.* 11. 6.
i. *hujus.*
h 1. 6.
k *conſtr.* (6.
l *prius.* & 19.
m 22. 6.
n *Nat.*
o 9. 5.
p 4. 6.
q 5. *def.* 6.
r *cor.* 20. 2 *hujus.* & 15. 5.
s *conſtr.*
t 11. 5.
u 41. 1. *hujus.*
x 2. *ax.*
a 1. 6.
b 4. 6.
c 11. 5.
d 1. 6.

Nat. D B × B E. B C × B H :: triang. D B E. triang. C B H.

Nam ducantur C V, D Q ad B H perpendiculares. Eſtque D Q × B E. D B × B E :: (D Q. D B :: C V. C B ::) C V × B H. C B × B H. & permutando D Q × B E [a] (hoc eſt 2 triang. D B E). C V × B H [a] (2 triang. C B H) :: D B × B E. C B × B H [d] :: triang. D B E. triang. C B H.

Prop. XVI.

Fig. 204. 205. 206

Si coni sectionem, vel circuli circumferentiam contingentes duæ rectæ lineæ (AC, BC) in unum conveniant (in C); & ab aliquo puncto (D) eorum, quæ sunt in sectione, ducatur linea (DF) uni (BC) contingentium æquidistans, quæ & sectionem, & alteram (AC) contingentium secet (in E, & F); ut quadrata contingentium (BC, AC) inter sese, ita erit rectangulum contentum lineis (FE, ED) quæ interjiciuntur inter sectionem & contingentem, ad quadratum lineæ (AE) inter æquidistantem (FE) & tactum (A) interjectæ.

a 46. & 47. 1.
b 6. 2. (*hujus*.
c 1. *hujus*.
d 16. 5. & 22. 6.
e 19. 5.
f *prius*.
g 2. *hujus*.
h 22. 6.

Per A, B ducantur diametri AGH, KBL; & DN ad AC parallela. [a] Liquet esse FK = KD, [b] unde FE × ED + DKq = EKq. Item CBq. triang CBL [c] (triang CAH) [d] :: EKq triang. EKL [d] :: DKq. triang DKN [e] :: EKq — DKq. triang EKL — triang DKN (hoc est [f] ::) FE × ED. 4lat DL ([g] = triang AEG). ergo permutatim CBq. FE × ED :: (triang CAH. triang AEG [h] ::) CAq AEq. vel iterum permutando CBq. CAq :: FE × ED. AEq. *Q. E. D.*

Not: 2 Coroll. venet in Coroll. 1 Coroll. Codit in 2 sectionem, in qua tres cæteras pro hypothesi non est applicata ad quæ cadit, seq. refertur eb in

Coroll. 1. FE × ED. CBq :: AEq. ACq.

2. { DKq. triang. DKN :: } FE × ED. 4lat. DL.
{ CBq. triang. CBL }

Prop. XVII.

Fig. 207. 208.

Si coni sectionem, vel circuli circumferentiam contingentes duæ rectæ lineæ (AC, BC) in unum conveniant (in C); sumantur autem in sectione duo quævis puncta (D, E), & ab iis ducantur lineæ (EFIK, DFGH) contingentibus æquidistantes), quæ & sibi ipsis, & * lineæ occurrant; ut quadrata contingentium (AC, BC) inter sese, ita erit rectangulum contentum lineis (KF, FE), quæ interjiciuntur inter sectionem, & linearum occursum (F), ad rectangulum quod lineis (HF, ED) similiter sumptis continetur.

* sectioni.

* Cor. 2. præc.
a 3. *hujus*.
b 16. 5. & 22. 6.
c 1. *hujus*.

Per A, B ducantur diametri AMLN, BXOP, & DX, EM tangentibus parallelæ. Estque KF × FE. 4lat FM [a] (vel 4lat. FX) * :: (EIq triang EIM [b] ::) ACq. triang ACN [c] (vel triang BCP). Similique discursu, HF × FE. 4lat FX :: CBq. triang BCP. & inverse 4lat FX. HF × FE :: triang. BCP. CBq.

ergo

ergo ex æquali KF × FE. HF × FD :: ACq. CBq. *Q. E. D.* Fig. 209.

Si punctum F intra sectionem cadat, similis est discursus, nisi quòd *pro.* 6. 2. adhiberi debeat 5. 2 *Elem.* (in præcedenti.)

Prop. XVIII.

Si oppositas sectiones (AB, MN) contingentes duæ rectæ lineæ (ACLE, BCH) inter se conveniant (in C); sumatur autem in quavis sectione (MN) aliquod punctum (D), & ab eo ducatur linea (DFGE) uni contingentium (BC) æquidistans, quæ & sectionem, & alteram contingentium secet in (F, & E): ut quadrata contingentium (BC, CA) inter sese, ita erit rectangulum contentum lineis (FE, ED), quæ interjiciuntur inter sectionem & contingentem (AE) ad quadratum lineæ (AE) inter æquidistantem (FE), & tactum (A) interjectæ. Fig. 210.

Ducatur DX ad AE parallela. Estque FE × ED + DOq[a] = OEq. ac BCq. triang BCL[c] (vel triang ACH)[b] :: (OEq. triang, OEL[b] :: DOq. triang DOX[d] :: FE × ED. 4lat. DL[e] (vel triang. AEG), itemque triang. ACH. ACq[b] :: triang. AEG. AEq. Ergo ex æquali BCq. ACq :: FE × ED. AEq. *Q. E. D.*

a 48. 1. *huj.* & 6. 2.
b 15. 5. *et* 22. 6.
c 1. *hujus.*
d 19. 5.
e 6. *hujus.*

Prop. XIX.

Si quæ oppositas sectiones contingunt duæ rectæ (AF, DF) in unum conveniant (in F), & ducantur contingentibus æquidistantes, (GHIKL, MNXOL), quæ & sibi ipsis & sectioni occurrant; ut quadrata contingentium (AF, FD) inter sese, ita erit rectangulum contentum lineis (GL, LI) quæ interjiciuntur inter sectionem, & linearum occursum, ad rectangulum, quod lineis (ML, LX) similiter sumptis continetur. Fig. 211.

Sint HAEC, NDEB diametri, & XR, IP tangentibus AF, DF parallelæ. Estq[illegible]q. triang AFS (vel triang DFT)[b] :: (HLq. triang HLO[b] :: HIq. triang HIP[b] :: GL[illegible] [illegible][d] (vel 4lat KX). itémque triang DFT. DFq[c] :: 4lat KX ML × LX. ergo ex æquo AFq. DFq :: GL × LI. ML × LX. *Q. E. D.*

a 4. *hujus.*
b 16. 5. *et* 22. 6.
c 2. *cor.* 16. *huj.*
d 7. *hujus.*

Prop. XX.

Si quæ oppositas sectiones (AB, CD) contingunt duæ rectæ lineæ (AF, CF) sibi invicem occurrant; & per occursum (F) ducatur linea Fig. 212.

nea (B F H D) tactus conjungenti (A C) æquidistans, quæ secet utramque sectionem (in B D); ducatur autem alia linea (G L S M N X) æquidistans eidem, sectionesque & contingentes secans: erit ut rectangulum contentum lineis (B F, F D), quæ inter occursum sectionum, & sectones interjiciuntur, ad quadratum lineæ contingentis (A F), ita rectangulum, quod continetur lineis (G L, L X) intersectiones, & contingentem interjectis, ad quadratum lineæ (A L) ad tactum abscissæ.

a 38. el. 39. 2. hujus.
b 45. 1. hujus.
c 16. 3. & 22.
d 19. 5. (6.
e 5. hujus.

Sint A E H, E F diametri; ducantúrque G P, B R ad A E parallelæ. Estque B Fq. [a] (BF × F D). triang B F R [b] (triang A F H) [c] :: L Sq. triang L S [d] :: G L × L X. 4lat G L F P [e] (triang A L M). Item triang A F H. A Fq [c] :: triang A L M. A Lq. ergo ex æquo B F × F D. A Fq :: G L × L X. A Lq. *Q. E. D.*

Coroll. Triang. B F R. B F × F D :: 4lat G L F P. G L × L X.

Prop. XXI.

Fig. 213.

Iisdem positis, si in sectione sumantur duo puncta (G, K), & per ipsa ducantur rectæ lineæ; una quidem (N X G O P, vel K S T) contingenti (A F) æquidistans; altera verò (G L M, vel K O V χ Ψ ω) lineæ (A C) tactus conjungenti æquidistans; quæ & sibi ipsis, & sectionibus occurrant: Erit ut rectangulum contentum lineis (B F, F D), quæ interjiciuntur inter occursum contingentium, & sectiones, ad quadratum contingentis (A F); ita rectangulum contentum lineis (K O, O ω) inter sectiones, & linearum occursum interjectis, ad rectangulum, quod lineis (N O, O G) similiter sumptis continetur.

a 41. 1. huj.
b 4[illegible]. et 22. 6
c cor. 16. huj.
d 12. hujus.
e cor. præced.

Nam A F q. triang A F H [a] (triang B Y F) [b] :: X O q. triang. X O Ψ [b] :: X G q triang X G M [c] :: N O × O G. 4lat G O Ψ M [d] (4lat K O R T). [e] Item triang. B Y F. B F × F D :: [e] 4lat K O R T. K O × O ω. ergo ex æquali A F q. B F × F D :: N O × O G. K O × O ω atque inversè.

[illegible]

Prop. XXII.

Fig. 214.

Si oppositas sectiones (A, B) contingant duæ rectæ lineæ (A C, B D) inter se æquidistantes; ducantur autem aliæ lineæ, quæ & sibi ipsis, & sectionibus occurrant (una quidem K E M) contingentibus æquidistans, altera verò (G X E) æquidistans ei (A B), quæ tactus conjungit: erit ut transversum latus ad rectum figuræ, quæ ad lineam (A B) tactus conjungentem constituitur; ita rectangulum contentum lineis

lineis (GE, XE) inter ſectionem, & linearum occurſum interjectis, ad rectangulum, quod lineis (KE, EM) ſimiliter ſumptis continetur.

Ducantur GF, XN ad AC parallelæ: hæ (& parallela his KM) ad [a] diametrum AB ordinatim applicantur: quare BL × LA. KLq [b] :: (ABR [b] :: BN × NA [c] (FA × NA). XNq. [d] :: BL × LA — FA × NA. KLq — XNq (— ELq) (hoc eſt) [e] :: FL × LN. [f] KE × EM. atqui FL × LN [g] = GE × EX. [h] ergo AB. R :: GE × EX. KE × EM. *Q. E. D.*

a 31. 2. *hujus.*
b 21. 1. *hujus.*
c *cor.* 16. 1. *hujus.*
d 19. 5.
e *Not.*
f 5. 2.
g 34. & *ſch.* 48. 1.

Not. [c] Quod FL × LN = BL × LA — FA × NA ſic patet. Biſecetur BA, vel FN in Z. Eſtque BL × LA + ZAq [k] = (ZLq [k] = FL × LN + ZNq [l] =) FL × LN + FA × AN + ZAq (nam ZNq [k] = BN × AN + ZAq). ergo BL × LA = FL × LN + FA × AN.

h 7, & 11. 5.
k 2. 6.
l 2. 02. 1.

Prop. XXIII.

Si in oppoſitis ſectionibus (AB, CD, EF, GH), quæ conjugatæ appellantur, duæ rectæ lineæ (AL, EL), contingentes oppoſitas ſectiones (AB, EF) conveniant in quavis ſectione (in L); ducantur autem aliquæ lineæ (GO, HS) æquidiſtantes contingentibus (AL, EL; quæ & ſibi ipſis, & aliis ſectionibus (CD, GH) occurrant: ut quadrata contingentium (AL, EL) inter ſeſe, ita erit rectangulum contentum lineis (GX, XO) quæ inter ſectiones, & occurſum (X) interjiciuntur ad rectangulum, quod lineis (HX, XS) ſimiliter ſumptis continetur.

Fig. 215.

Ducantur ST ad AL, & OY ad EL parallelæ. Eſtque HP × PS, & GM × MO, ac ELq. triang EVL. [a] (triang. ALχ) [b] :: PXq. triang PNX [c] :: HX × XS. 4lat TNXS [d] (XRYO). Item triang. ALχ. ALq [c] :: 4lat. XRYO. GX × XO. ex æquo igitur ELq. ALq :: HX × XS. GX × XO. Quod *E. D.*

a 4 *hujus.*
b 16. 5. *et* 22.
c *ut ſæpe ſupra.*
d 19. 6.

Prop. XXIV.

Si in oppoſitis ſectionibus (A, B, C, D) quas conjugatas appellamus, à centro (E) ad ſectiones ducantur duæ lineæ (AC, DB) quarum una quidem (AC) ſit tranſverſa diameter, altera verò (DB) recta; & ducantur aliæ lineæ (FL, MR) his diametris æquidiſtantes, quæ & ſibi ipſis, & ſectionibus occurrant, ita ut occurſus (X) ſit in loco inter quatuor ſectiones intermedio: rectangulum contentum portionibus (FX, XL) lineæ diametro tranſverſæ æquidiſtantis, unà cum

Fig. 216. 217.

cum eo, ad quod rectangulum ex portionibus (M X, X R) lineæ æquidistantis rectæ diametro proportionem habet eandem, quam diametri rectæ (D B) quadratum ad quadratum transversæ (A C); æquale erit duplo quadrati, quod à dimidia (A E) transversæ diametri constituitur.

1 *Not.* R N * M N + N X * P X = R X * X M. Nam N X * P X + O Xq [o] = O Nq. [p] ergo N X * P X + O Xq + R N * N M = (O Nq + R N * N M [o] = O Mq =) R X * X M + O Xq. unde (auferendo O Xq utrinque) [q] erit N X * P X + R N * N M = R X * X M.

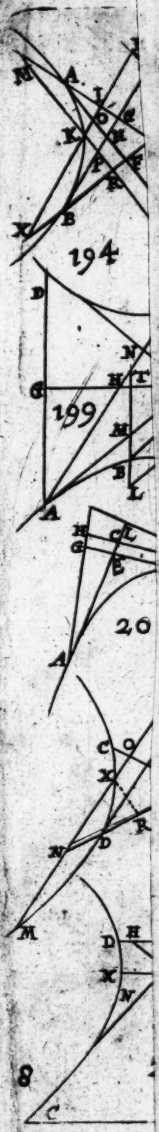
2 *Not.* L X * X F + X H * X K = F K * H F (vel L H * H F) Nam K X * X H + I Hq [r] = I Xq. [s] quare K X + X H + I Hq + L X * X F = (I Xq + L X * X F [t] = I Fq [t] =) L H * H F + I Hq. & (auferendo commune I Hq) [u] erit K X * X H + L X * X F = L H * H F.

Sint S E T, V E Y asymptoti; sitque occursus primò in angulo S E V, vel S E Y. Per A ducatur tangens S A V. Estque D Bq. A Cq [a] :: (D E q. A E q [b] (hoc est) :: S A * A V. A E q [c] = S A. A E + A V. A E (hoc est) [d] :: N X. X H + P X. X K [c] =) N X * P X. X H * X K [c] :: D E q + N X * P X. A E q + X H * X K [f] (hoc est ::) P M * M N [g] (vel R N * M N) + N X * P X. [f] L H * H F [g] (vel F K * H F) + X H * X K (hoc est) [h] ::) R X * X M. F K * H F + X H * X K [g] ::) D Bq. A Cq. atqui L X * X F + X H * X K [i] = F K * H F [f] = A E q. ergo L X * X F + L H * H F + X H * X K = 2 A E q. *Q. E. D.* Sin occursus H sit in asymptoto, erit F H * H L [f] = A E q. [f] & M H * H R = D E q unde D E q. A E q [n] :: M H * H R. F H * H L. & 2 F H * H L [m] = 2 A E q.

Prop. XXV.

Fig. 219. Iisdem positis, sit linearum ipsis A C, B D æquidistantium occursus in una sectionum (D B) atque in puncto X, ut positum est: Dico rectangulum (O X N) contentum portionibus lineæ, quæ transversæ diametro (A C) æquidistat, majus esse, majus est quam illud, ad quod rectangulum (R X M), ex portionibus lineæ æquidistantis rectæ diametro (D B), eandem proportionem habet, quam rectæ diametri quadratum ad quadratum transversæ, duplo quadrati ejus, quod à dimidia transversæ diametri constituitur.

Nam D E q [x] (id est P M * M H). E A q [x] (L K * K S) :: [b] P X * X H.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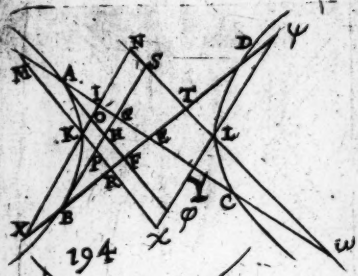

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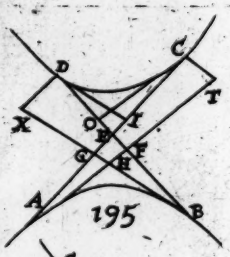

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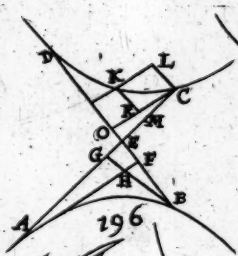

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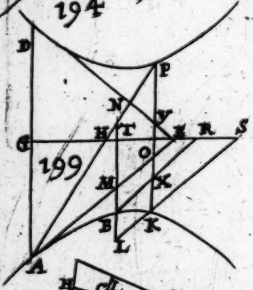

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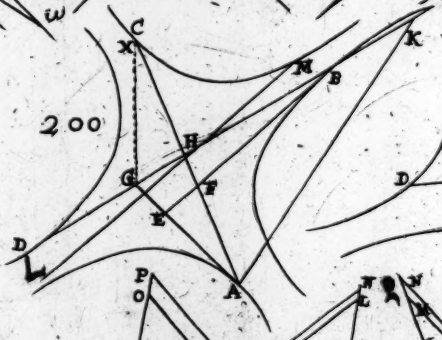

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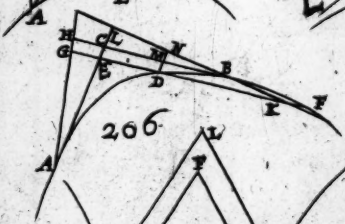

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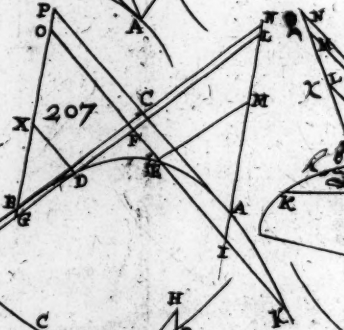

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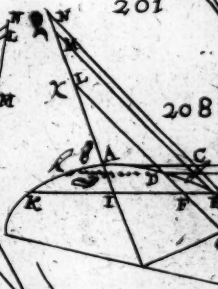

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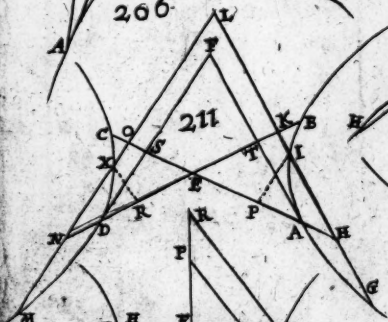

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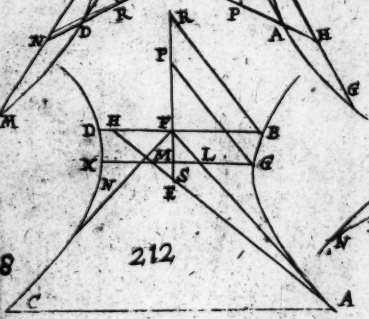

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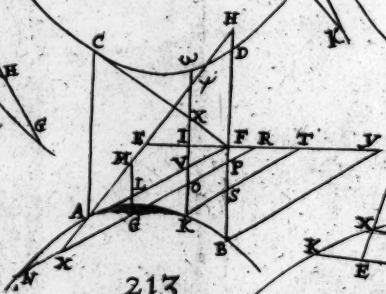

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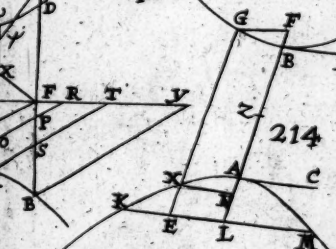

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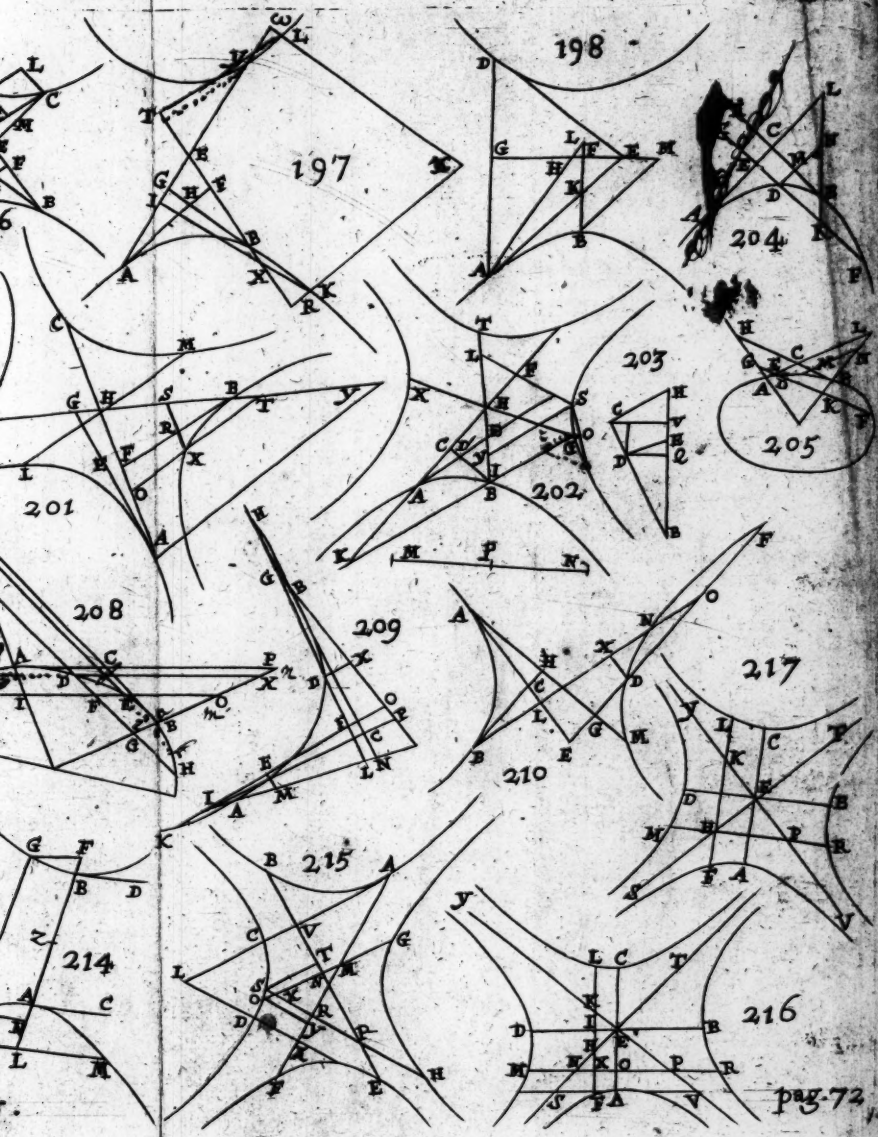
Apollonius.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XH. SX × XL [c] :: RX × XM. TX × XK :: DEq. EAq; [d] atqui OT × TN = 2AEq. [c] ideóque OX × XN [c] (hoc est TX × XK + OT × TN) = TX × XK + 2AEq. *Q. E. D.*

Not. * PX × XH — PM × MH = RX × XM. & × SX × XL — LK × KS = TX × XK.

Not. OX × XN [z] = TX × XK + OT × TN.

c 19. 5. *Not.* 12.
d 23. 2 *hujus.*
e 2. *ax.* 1.
f *Not.*
* *vid. Not. ad* 22 *hujus.*
x *v. not. ad* 24 *b.*
z *v. not. ad* 24 *b.*

Prop. XXVI.

Quòd ſi æquidiſtantium occurſus ad punctum X ſit in una ſectionum AC, ut poſitum eſt; rectangulum (LXF), quod continetur portionibus lineæ æquidiſtantis tranſverſæ diametro (AC), minus erit quàm illud, ad quod rectangulum (RXG), portionibus alterius lineæ contentum, eandem proportionem habet quàm rectæ diametri quadratum, ad quadratum tranſverſæ, duplo quadrati ejus, quod à dimidia tranſverſæ diametri conſtituitur. Fig. 220.

Nam (ſimili ratione) DEq. [a] (hoc eſt VG × GS, [b] vel RS × SG). AEq :: VX × XS. KX × XH [c] :: RS × SG + VX × XS. AEq + KX × XH [d] (hoc eſt) :: RX × XG. AEq + KX × XH. Atqui AEq [e] = (LH × HF =) KX × XH — LX × XF. & [f] proinde 2AEq + LX × XF = KX × XH + AEq.

a 11. 2 *hujus.*
b 8. 2. *hujus.*
c 12. 5.
d *Not.*
e 11. 2. *huj.*
f 2 *ax.* 1. *huj.*

Not. 1.* RS × SG + VX × XS = RX × XG.
2. [g] LH × HF = KX × XH — LX × XF.

* *vid. Not.* 2. *ad* 24. *hujus.*
g *v. not. ad* 22. *hujus.*

Prop. XXVII.

Si in Ellipſi vel circuli circumferentia ducantur conjugatæ diametri (AC, BD), quarum altera quidem (AC) ſit recta, altera verò (BD) tranſverſa: & ducantur duæ rectæ lineæ (KM, NH) diametris æquidiſtantes, quæ & ſibi ipſis, & ſectionibus occurrant: quadrata ex portionibus (NF, FH) lineæ æquidiſtantis tranſverſæ diametro, quæ inter ſectionem, & linearum occurſum interjiciuntur, aſſumentia figuras (KF × Z, & FM × Y) ex portionibus (KF, FM) lineæ, quæ rectæ diametro æquidiſtat, inter linearum occurſum (F), & ſectionem interjectis, ſimiles & ſimiliter deſcriptas ei, quæ ad rectam diametrum conſtituitur, quadrato tranſverſæ diametri æqualia erunt. Fig. 221.

Ducatur NX ad AE parallela. Sintque R, S recta latera pro diametris

metris BD, AC; [a] itaque NX.V :: AC.S :: KL.X. Et propter R, AC, BD, S ÷÷ [b] erit R.BD [c] hoc est NXq.DX×XB) :: AC.S [c] :: NX.V [f] :: NXq.NX×V. Unde NX×V [g] = (DX×XB [h] =) BEq — XEq (— NGq). Simili discursu KL×X = BEq — FGq. [k] ergo NX×V + KL×X + NGq + FGq = 2BEq. [l] atqui 2NGq + 2FGq = HFq + FNq. & 2NX×V + 2KL×X (hoc est 2FL×V + 2KL×X) [m] = FM×Y + KF×Z. [n] ergo HFq + FNq + FM×Y + KF×Z = 4BEq [o] = BDq. *Q. E. D.*

a 12. 6.
b 16. 5.
c 21. 1. *hujus.*
e *constr.*
f 1. 6.
g 9. 5.
h 5. 2.
k 2. *ax.* 1.
l 9. 2.
m *ex.* 9. 2.
n 2 *ax.* 1.
o 4. 2.

Prop. XXVIII.

Fig. 222. Si in oppositis sectionibus (A, B C, D), quas conjugatas appellamus, ducantur diametri conjugatæ (A C, B D), ut earum altera (AC) recta sit, altera (B D) transversa; & ducantur duæ rectæ lineæ (FK, L N) diametris æquidistantes, quæ & sibi ipsis, & sectionibus occurrant: Quadrata ex portionibus (L G, G N) lineæ æquidistantis rectæ diametro (A C), quæ inter linearum occursum, & sectiones interjiciuntur, ad quadrata ex portionibus (FG, GK) alterius lineæ, quæ transversæ diametro (B D) æquidistant, inter sectiones, & occursum linearum interjectis, eandem proportionem habent, quam rectæ diametri quadratum ad quadratum transversæ.

Ordinatim applicentur FO, & LX. Estque AFq. EBq :: * AC q. BDq [a] :: R.BD :: FOq (EHq). DO×OB [b] :: CX×XA. LXq (EMq) :: AEq — CX×XA + EHq. EBq + DO×OB + FMq [c] (hoc est) :: XEq + EHq. OEq + EMq. [e] (hoc est) :: LMq. + GMq. FHq + GHq [e] :: 2LMq + 2GMq. 2FHq + 2GHq [f] (hoc est) :: LGq + GNq. FGq + GKq [g] :: ACq. BDq. *Q. E. D.*

a 15. 5.
a *cor.* 19. 6.
b 21. 1. *hujus.*
c 12. 5.
d 6. 2.
e 15. 5.
f 9. 2.
g 11. 5.

Lemma pro seq.

Fig. 223. Sit linea recta composita $a + b + c + a$. Erit Quad. $a + b +$ Quad. $c + a = bb + cc + 2a \times : b + c + a$. hoc est $aa + bb + 2ba + cc + aa + 2ca = bb + cc + 2ba + 2ca + 2aa$.

Prop. XXIX.

Fig. 224. Iisdem positis, si linea (L N) rectæ diametro æquidistans, secet asymptotos; quadrata ex ipsius portionibus (X G, G O), quæ inter linearum occursum, & asymptotos interjiciuntur, assumentia dimidium quadrati facti à recta diametro, ad quadrata ex portionibus (FG, GK,)

G K) lineæ (F K) quæ transversæ diametro æquidistat, inter occursum linearum, & sectiones interjectis, eandem proportionem habent, quàm rectæ diametri quadratum, ad quadratum transversæ.

Nam ob L X [a] = O N, erit L Gq + G Nq [b] = X G q + G Oq + 2 L X × X N [c] (+ 2 A Eq). er[illegible] X G q + G Oq + 2 A Eq (+ ½ A Cq). F Gq + G Kq [d] :: L Gq + G Nq. F Gq + G Kq [e] :: A Cq. B Dq. *Q. E. D.*

a 16. 2. *hujus.*
b *lemm. præc.*
c 10. 2. *hujus.*
d 7. 5.
e 28. *hujus.*

Prop. XXX.

Si hyperbolen contingentes duæ rectæ lineæ (A D, C D) sibi ipsis occurrant; & per tactus (A, C) producatur linea (A C); per occursum verò (D) ducatur linea (DL) æquidistans uni (F E), asymptoton (F E, FG), sectionémque & lineam tactus conjungentem secans, bifariam dividetur (in K). Fig. 225.

Jungatur F D B M; sitque F H = F B: ducantúrque B E, K N ad A C parallelæ. Estque D Nq. N Kq [a] :: (F Bq. B Eq [b] :: H B. R [c] ::) H N × N B. N Kq. [d] unde H N × N B = D Nq, [e] item M F × F D = F Bq. Ergo D Nq + M F × F D [f] = (H N × N B + F Bq. [g] =) F Nq ergo D N = N M. [h] quare D K = KL. *Q. E. D.*

Coroll. F Bq. B Eq :: H N × N B. N Kq.

a 4. & 22. 6.
b *cor.* 1. 2. *huj.*
c 21. 1. *hujus.*
d 9. 5.
e 37. 1. *hujus.*
f 2. *ax.* 1.
g 6. 2.
h *conv.* 6. 2.
k 2. 6.

Prop. XXXI.

Si quæ oppositas sectiones contingunt duæ rectæ lineæ (A C, B C) sibi ipsis occurrant; & per tactus (A, B) linea (A B) producatur; per occursum verò (C) ducatur linea (C H) æquidistans asymptoto (EF); quæ sectionem, & lineam tactus conjungentem secet; linea (C H) inter occursum, & eam quæ tactus conjungit interjecta à sectione bifariam dividetur (in G). Fig. 226.

Not. E *est centrum.*

Ducantur rectæ C E D, & tam E K M N & G X ad A B, quàm K F, G L ad C D parallelæ. Estque N L × L K. L Gq [a] :: (E Kq. K Fq [b] ::) M Lq. L Gq. [c] unde N L × L K = M Lq. [d] quare M Lq + K Eq = (N L × L K + K Eq [e] = L Eq [f] =) G X q. ergo cùm G X q. M Lq + K Eq :: X C q. L G q + K Fq; [h] erit X C q = (L Gq + K Fq =) [f] (E Xq) [k] + C E × E D. [l] ergo C X = X D; [m] adeóque C G = G H.

a *cor. præc.*
b 4. & 22. 6.
c 9. 5.
d 2. *ax.* 1.
e 6. 2.
f 34. 1. [illegible]
h 14. 5.
k 38. 1. *huj.*
l *conv.* 5. 2.
m 2. 6.

Prop. XXXII.

Fig. 227.

Si hyperbolen contingentes duæ rectæ lineæ (A F, C F) ſibi ipſis occurrant; & per tactus (A, C) producatur linea (A C); per contingentium verò occurſum (F) ducatur linea (F K) tactus conjungenti (A C) æquidiſtans; & per punctum (H), quod conjungentem tactus (A C) bifariam ſecat, ducatur linea (H K) æquidiſtans aſymptoton alteri (D E): quæ (H K) inter dictum punctum (H), & lineam æquidiſtantem (F K) incerjicitur, à ſectione bifariam dividetur (in L).

a cor. 30. *huj.*
b 4 & 22. 6.
c 9. 5.
d 37. 1. *hujus.*
e 2. *ax.* 1.
f 6. 2.
g *Nos.*
h 2. 6.
k 1. 2.
l 2. *ax.* 1.
m *prius.*
n 2. 2.
o 3. *ax.* 1.
p *conſtr.*
q 3. 2.
r 1. *ax.* 1.
s 16. 6.
t 9. 5.
u *conſtr.*

Ducantur B E, L M ad A C parallelæ. Eſtque G M × M B : M Lq. [a] :: (D B q. B Eq [b] ::) H M q. M Lq. [c] unde G M × M B = H Mq. [d] Item H D × D F = D Bq. [e] ergo H Mq + H D × D F = (DBq + GM × MB [f] =) DMq. [g] ergo FM = MH. [h] & propterea KL = LH.

Nat. Fiat D Z = M H. Eſtque M D × D F + M H × DF [k] = H D × D E. [l] ideóque (addendo utrique ipſum. M H q.) M D × D F + M H × D F + M H q = (H D × D F + M H q [m] = D Mq [n] =) M D × D F + M D × F M. ergo (ablato communi M D × D F) M D × F M [o] = M H × D F + M Hq [p] = D Z × D F + D Z q. q = F Z × Z D [r] = M D × F M. [s] quare F Z. F M :: M D. Z D. & componendo Z M. F M :: Z M. Z D. unde E M [t] = (Z D [v] =) H M.

Prop. XXXIII.

Fig. 228.

Si, quæ oppoſitas ſectiones contingunt, duæ rectæ lineæ (A G, D G), ſibi ipſis occurrant, & per tactus (A, D) linea (A D) producatur; per contingentium verò occurſum (G) ducatur linea (C F) æquidiſtans tactus conjungenti (A D); & per punctum (L), quod conjungentem tactus (A D) bifariam ſecat, ducatur linea (L N) æquidiſtans aſymptoton alteri (H K), conveniénſque cum ſectione, & cum linea æquidiſtante (F C) per occurſum (G) ducta: quæ (L N) inter dictum punctum (L) & lineam æquidiſtantem (F N) interjicitur, à ſectione bifariam dividetur (in M).

a 4. & 22. 6.
b cor. 30. *huj.*
c 12. 5.
d [illegible] 2.
e [illegible] 1. *huj.*
f 3. 1. &c.
g 2. 5.
k *coroll.* 5. 2.
l 2. 6.

Ducantur M P ad A D, & E K, M X ad G H parallelæ. Eſtque M Pq. P Lq [a] :: (H Eq. E Kq [b] :: E X × X B. X Mq [c] :: + H Eq + E X × X B. E Kq + X Mq (hoc eſt) :: [d] H Xq (vel P Mq) [e] G H × H L + X Mq (vel) ::) [f] P Mq. G H × H L + H Pq. ergo P Lq [g] = G H × H L + H Pq. [h] quare L P = P G. & [k] conſequenter L M = M N.

Prop. XXXIV.

Si in una (DC) aſymptoton (DC, DE) hyperbolæ, ſumatur aliquod punctum (C); ab eóque recta linea (CE) ſectionem contingat; & per tactum (B) ducatur æquidiſtans (FG) aſymptoto (DC); quæ (CG) per dictum punctum (C) tranſit, alteri (DE) aſymptoton æquidiſtans, à ſectione bifariam dividetur (in A.) Fig. 229.

Ducantur AH ad CD, & BK ad DE parallelæ; éſtque $CB^2 = BE$; [b] ideóque $CK = KD$. Item $KB \times KD$ [c] $= CA \times CD$, [d] hoc eſt $CG \times CK = CA \times CD$. [e] quare CG. CA :: (CD. CK [f] ::) 2. 1. *Q. E. D.*

a 3. 2 *hujus.*
b 2, 6.
c 12. 2 *hujus.*
d *Sch.* 48. 1.
e 15. 6.
f *prius.*

Prop. XXXV.

Iiſdem poſitis, ſi à ſumpto puncto (C) ducatur recta linea (CF) ſectionem ſecans in duobus punctis (A, F); erit ut tota (FC) ad eam (CA), quæ extra ſumitur, ita inter ſeſe portiones (FL, LA) illius (FA), quæ intra ſectionem continetur. Fig. 230.

Ducantur CNX, KAVM, OPBR, YF parallelæ ad DE; & APS, TFRMX ad CD parallelæ. Eſtque $AC^2 = FG$, ac idcirco TG [b] $= (KA^c =)$ DS. & CK [b] $= (TF^c =)$ DY. unde DK [d] $= CY$. Item rectang HK. rectang KN [e] :: (DK. KC [f] :: CY. KC [g] :: FC. CA [g] :: MK. KA :: [e] rectang MD. rectang AD (hoc eſt) [h] ::) rectang MD. rectang DB [k] (vel rectang ON) [l] :: rect. MD — rectang HK. rectang QN — rectang KN (hoc eſt) :: rect. MH. rectang KB :: rectang MH. rectang AH (ob rectang KS [b] $=$ rectang HO; & AB commune :: MV. VA [m] :: FL. LA [n] :: FC. CA. *Q. E. D.*

a 8. 2 *hujus.*
b 46. & 14. 5.
c 34. 1.
d 2 ax. 1.
e 1. 6.
f 7. 5.
g 4. 6.
h 12. 2 *hujus.*
k 3. 2 *hujus.*
l 19. 5. (*et* 4. 6.
m 2 & 3. *ax.* 1.
n 11. 5.

Prop. XXXVI.

Iiſdem poſitis, ſi à puncto (G) ducta linea (HG) neque ſectionem in duobus punctis ſecet, neque æquidiſtans ſit aſymptoto (CE), ſed cum oppoſita ſectione conveniat (in A); erit ut tota (AK) ad lineam (KH), quæ inter ſectionem (H), & æquidiſtantem (KL) per tactum (B) interjicitur; ita quæ (AG) eſt inter oppoſitam ſectionem (A) & aſymptoton (CG) ad eam (GH), quæ inter aſymptoton (CG), & alteram ſectionem (H). Fig. 231.

Ducantur

Ducantur H M, A N ad C G parallelæ; & B X, G P, R H S N parallelæ ad D E. Estque rectang N C. rectang C H [a] :: (N S. S H :: [b] A G. G H [c] :: D H. G H. (ob A D = [d] G H) [b] :: C S. S G [e] :: rect C R. rectang R G [f] :: rectang N C + rectang C R. rectang C H [f] (vel rectang L X [g] vel rectang B G) + rectang R G (hoc est) :: rectang. N L. rectang R X (hoc est [h] ::) rectang N L. rectang L H :: [a] N R. R H :: [b] A K. K H :: [k] A G. G H. *Q. E. D.*

a 1. 6.
b 4. 6.
c 7. 5.
d 16. 2 *hujus.*
e 12. 6.
f 12. 2 *hujus.*
g 3. 2 *hujus.* & 4. 6.
h *Not.* & 7. 5.
k 11. 5.
l 12. 2 *hujus.*

Not. Rectang R X = rectang L H. ob rectang X H [l] = rectang. M B. & commune rectang B H.

Prop. XXXVII.

Fig. 232. 233. 234.

Si coni sectionem, vel circuli circumferentiam, vel sectiones oppositas, contingentes duæ rectæ lineæ (A C, B C) sibi ipsis occurrant, & per tactus (A, B) producatur linea (A B); à contingentium verò occursu (C) ducatur linea (C F) sectionem secans in duobus punctis (D, F); erit ut tota (F C) ad eam (C D) quæ extra sumitur, ita portiones (F E, E D) inter sese, quæ à linea (A B) tactus conjungente fiunt.

Ducantur diametri C H, A K; & rectæ D P, F R ad A C parallelæ, & L F M, N D O parallelæ ad A B.

Estque triang L M C. triang X O C :: [a] L Mq. X Oq [a] :: L Cq. C Xq. F Cq. C Dq [a] :: F Mq. D Oq [a] :: triang F R M. triang D P O :: triang L A K ([b] hoc est 4lat. L C R F). triang X A N ([b] hoc est 4lat. X C P D) [a] :: L Aq. A Xq [c] :: F Eq. E Dq [d] :: F Cq. C Dq. [e] quare F E. E D :: F C. C D. *Q. E. D.*

a 4. & 22. 6.
b *huj.* 21. 2. *ax.* 1.
c 2. & 22. 6.
d 11. 5.
e 22. 6.

Coroll. L M. X O :: F E. E D.

Nob: at claring poteat quod per totam 9. 6. supponendum est contingentem aa transire per punctum diametrum

Prop. XXXVIII.

Fig. 235. 236.

Iisdem positis, Si per contingentium occursum (C) ducatur recta linea (C O) æquidistans tactus conjungenti (A B); & per punctum (E), quod conjungentem tactus bifariam dividit, ducatur linea (F O) secans, & sectionem ipsam in duobus punctis (F, D), & lineam æquidistantem (C O) per occursum ductam: erit ut tota (F O) ad eam (O D), quæ extra sumitur inter sectionem, & lineam æquidistantem; ita portiones (F E, E D) inter sese, quæ à linea (A B) tactus conjungente efficiuntur.

Ducantur L F K M, D H G X N parallelæ ad A B; ac F R, G P ad F C parallelæ. Estque F E. E D [a] :: L M. X H [b] :: L C. C X [c] :: F O. O D [d] :: F E. E D. *Q. E. D.*

a *cor. præc.*
b 4. 6.
c 2. 6. & 11. 5.

Prop.

Prop. XXXIX.

Si, quæ oppoſitas ſectiones contingunt, duæ rectæ lineæ (AD, BD) ſibi ipſis occurrant, & per tactus (A, B) linea (AB) producatur; à contingentium verò occurſu (D) ducta linea (EG) & utramque ſectionem (E, F) & lineam (AB) tactus conjungentem ſecet (in G): erit ut tota (EG) ad eam (FG), quæ extra ſumitur inter ſectionem & conjungentem tactus; ita portiones (ED, DF) inter ſeſe, quæ inter ſectiones (E, F), & contingentium occurſum (D) interjiciuntur. Fig. 237.

Per centrum C ducatur ACK; & fiant EHSK, FNMXO ad AB parallelæ, ac EPFR ad AD parallelæ. Eſtque EH. HD [a] :: FM. MD. & HD. HS [a] :: MD. MX. unde ex æquo EH. HS :: FM. MX. [b] quare EHq. FMq [c] (id eſt triang. EHP. triang FMR) :: HSq. MXq [c] (hoc eſt triang DHS. DMX.) atqui triang EHP [d] = triang ASK + triang HDS. & triang FMR [d] = triang. AXN + triang DMX. [e] quare triang HDS. triang DMX (hoc eſt HDq. DMq. [e] vel EDq. DFq) :: triang ASK. triang AXN [f] :: KAq. ANq. Eſt autem KA. AN :: EG. GF (Nam KA. AQ. [c] :: EG. GQ. & AQ. AN [c] :: GQ. GF; adeóque ex æquo KA. AN :: EG. GF). [g] ergo demum eſt EG. GF :: ED. DF. Q. E. D.

a 4. 6.
b 16. 5. & 22. 6.
c 4. 6. & 22. 6.
d 11. *hujus*.
e 19. 5.
f 22. 6.
g 11. 5.

Not. KA. AN :: EG. GF.

Prop. XL.

Iiſdem poſitis, ſi per contingentium occurſum (D) ducatur recta linea (FG) tactus conjungenti (AB) æquidiſtans; & à puncto (E), quod conjungentem tactus bifariam dividit, ducatur linea (HL) ſecans utramque ſectionem (in H, K) & æquidiſtantem (FG in L) ei, quæ tactus conjungit: erit ut tota (HL) ad eam (LK), quæ extra ſumitur, inter æquidiſtantem, & ſectionem, ita portiones (HE, EK) inter ſeſe, quæ inter ſectiones, & conjungentem tactus interjiciuntur. Fig. 238.

Ducantur ACXT; & parallelæ HNMX, KO ad AB; & HR, KS ad AD. Eſtque triang HRN. [a] (triang XMA + triang MND). triang KSO [a] (triang AYP + triang POD) [b] :: HNq. KOq :: MAq. APq (nam HN. KO [c] :: HE. EK [d] :: XA. AY :: [c] MA. AP) [b] :: triang XMA. triang YAP [e] :: triang MND. triang

a 11. *hujus*.
b 4 & 22. 6.
c 4. 6.
d 34. 1. & 7. 5.
e 19. 5.

triang P O D[b] :: M Nq. P Oq[b] :: N Dq. D Oq[f] :: H Lq. L Kq[g] :: H Eq. E Kq. [h] quare H L. L K :: H E. E K. Q. E. D.

Not. N D. D O :: H L. L K. Nam N D. D O[k] :: (M D. D P[l] :: H V. V Q[k] ::) H L. L K.

f *Not.*
g *ſupra, et* 11. 5
h 22. 6.
k 4. 6.
l 34. 7. & 7. 5.

Prop. XLI.

Si parabolen contingentes tres rectæ lineæ (A E, C E, D F) inter ſe conveniant, in eandem proportionem ſecabuntur. (E D. D A :: C F. F E :: F B. B D).

Fig. 239. 240.

Ducatur A C, quam biſecet E G. Si hæc per tactum B tranſit, parallela, & [a]erit diameter. & [b]E B = B G. & D F ad A C [c] parallela. [d] Unde E D = D A, & E F = F G. [e] & D B = B F, quare liquet propoſitum.

a 29. 2. *hujus.*
b 35. 1. *huj.*
c 5. 2. *hujus.*
d 4. 6.
e 4. 6.

Sin per aliud punctum H tranſeat, per H ducatur tangens K H L ad A C parallela, & diameter M B X (per B) ad E G * parallela; & ordinatim applicentur A O, C P (ab A, & C). Eritque M B[f] = B P. ideóque M F[g] = F C. & E L[g] = L C (ob E H[f] = H G). unde F C. L C[h] :: (M F. E L[k] :: M C. E C) X C. G C. item L C. C E :: (1. 2 ::) G C.[l] C A. ergo ex æquali F C. C E :: X C. C A. quare inverſè dividendo F C. F E :: (C X. X A ::) E D. D A (nam K A. E A[m] :: 1. 2 :: B O. O N[n] :: D A. A N, & permutando K A. A D :: (E A. A N[o] ::) G A. A X. & E A. K A :: C A. G A. unde ex æquali E A. A D :: C A. A X. dividendóque E D. A D :: C X. X A). Porro, C X. X A[o] :: C P. A O[p] :: ½ C P. ½ A O (hoc eſt) [q] :: B F. D B. ergo E D. A D :: B F. D B.

* 46. 1. *huj.*
f 35. 1. *huj.*
g 2. 6.
h 7. 5.
k 15. 5.
l *conſtr.*
m *prius.*
n 2. 6.
o 4. 6.
p 15. 5.
q *prius* & 4. 6.

Prop. XLII.

Si in hyperbola, vel ellipſi, vel circuli circumferentia, vel oppoſitis ſectionibus, ab extremo diametri (A B) ducantur lineæ (A C, B D) æquidiſtantes ei, quæ ordinatim applicata eſt; & ducatur alia quæpiam linea (C E) quomodocunque contingens; abſcindet ex ipſis lineas (A C, B D) continentes rectangulum æquale quartæ parti figuræ, quæ ad eandem diametrum (A B) conſtituitur.

Fig. 241. 242. 243.

Per centrum F ducatur F G H ad A C vel B D parallela. In hac eſt [a]diameter (puta F G) ipſi A B conjugata. Item per tactum E ducatur E L ad A C parallela; & E M parallela ad A B. Eſtque K F. A F

a 16. 1. *huj.*

AF [b]:: AF. FL. unde (in hyperbole) KF. AF. [c] (KF + AF. AF + FL (hoc eſt) :: KA. AL. vel (in ellipſi) per converſam rationem, & permutando KF. AF (vel FB) :: KA. AL. [d] quare KB. (FB — KF. vel FB + KF) KF :: KL. (AL — KA vel AL + KA). KA. [e] hoc eſt BD. FH :: LE. AC. unde BD × AC [f] = (FH × LE [b] = FH × FM = [a] FGq [k] =) ¼ TR [l] = BD × AC. *Q. E. D.*

b *cor.* 37. 1. *b.*
c 12. 5.
d *inversè & converſè. vel componendo.*
e 4. 6.
f 16. 6.
g 34. 1. h 38. 1. *buj.* k *cor. def. ad* 16 1. *hujus.* l 1. *ax.*

Prop. XLIII.

Si hyperbolen contingat recta linea (CH), abſcindet ex aſymptotis (DC, DE) ad ſectionis centrum (D) lineas (DC, DH) continentes rectangulum æquale ei, quod continetur lineis (DF, DG) abſciſſis ab altera contingente (FG), ad ſectionis verticem (B), qui eſt ad axem (BD). Fig. 244.

Ducantur AK, BL ad DG; & AM, BN ad CD parallelæ. Eſtque CH [a] = 2AH; & ideò CD [b] = 2AM. & DH [b] = 2AK. unde CD × DH [c] = (4AK × AM [d] =) 4BL × BN. Simili diſcurſu 4BL × BN = FD × DG. [e] unde CD × DH = FD × DG. *Q. E. D.*

a 3. 2 *hujus.*
b 4. 6.
c *Sch.* 48. 1.
d 12. 2 *hujus.*
e 1. *ax.* 1.

Non aliter argumentabimur, etſi DB non ſit axis, ſed alia quæpiam diameter.

Prop. XLIV.

Si quæ hyperbolen, vel oppoſitas ſectiones contingunt, duæ rectæ lineæ (CF, EG) occurrant aſymptotis (DC, DE); quæ (CE, GF) ad occurſus ducuntur, lineæ (AB) tactus (A, B) conjungenti æquidiſtantes erunt. Fig. 245. 246.

Nam ob CD × DF [a] = ED × DG, [b] erit CD. ED :: DG. DF. [c] quare CE, GF parallelæ ſunt. ergo HG. GE :: HF. FC. item GE. GB :: [d] 2. 1 :: FC. CA. ergo ex æquo HG. GB :: HF. FA. inverſéque GB. HG :: FA. HF. & diviſè HB. HG :: HA. HF. [e] quare GF, & AB parallelæ ſunt. *Q. E. D.*

a 43. *hujus.*
b 15. 6.
c 6. 6. *&c.*
d 3. 2. *hujus.*

Prop. XLV.

Si in hyperbola, vel ellipſi, vel circuli circumferentia, vel oppoſitis ſectionibus, ab extremo axis (AB) lineæ (AC, BD) ad rectos angulos ducantur, & quartæ parti figuræ æquale rectangulum (AFB, AGB) comparetur ad axem ex utraque parte; quod in hyperbola quidem, & ſectionibus oppoſitis excedat figurâ quadratâ, in ellipſi ve- Fig. 247. 248.

rò deficiat; & ducatur linea (E C) sectionem contingens, occurrénsque eis (A C, B D) quæ sunt ad rectos angulos; lineæ (C F, D G), quæ ab occursibus (C, D) ducuntur ad puncta (F, G) ex eo comparatione factâ, angulos rectos (C F D, D G C) ad ea (F, G) efficient.

Nam A C * B D [a] = ($\frac{1}{4}$ TR [b] =) A F * F B. unde A C. A F [c] :: F B. B D. item anguli C A F, F B D [b] recti sunt. [d] ergo ang. A C F = ang B F D. [d] & ang. A F C = ang. F D B. ergo cùm anguli A C F, A F C [e] conficiant unum rectum, etiam anguli B F D, A F C uni recto æquabuntur. unde (in ellipsi) reliquus D F C [f] rectus erit. Simili discursu angulus C G D rectus ostendetur.

a 42. *hujus.*
b *hyp.*
c 15. 6.
d 6. 6.
e 32. 1.
f 2 *cor.* 13. 1.

Cor: A C F = Ang. B F D

Prop. XLVI.

Iisdem positis, lineæ conjunctæ æquales facient angulos (A C F, D C G, & C D F, B D G) ad contingentes (C D, B D).

Fig. 249. 250.

Circulus enim diametro D C descriptus [a] per puncta F, G transibit. unde ang. D C G [b] angulo D F G, [c] hoc est angulo A C F, æquatur. Similiter ang. C D F = ang. B D G. *Q. E. D.*

a *prac.* 31. 3.
b 21. 3.
c *in praced.*

Prop. XLVII.

Iisdem positis, linea (H E) ab occursu (H) conjunctarum (C G, F D) perpendicularis est ad contingentem (C D).

Fig. 251. 252.

Si negas, esto H L ad E C perpendicularis; & ordinatim applicetur E M (ad B D parallela). Atque ob ang. G D B [a] = (ang. C D F [b] =) ang. L D H. & [c] rectos D B G, D L H, erunt trigona G D B, D H L similia. quare B D. D L [d] :: (G D. D H [d] :: F C. C H (ob similia trigona G D H, F C H; nam anguli [e] C F H, D G H recti sunt, & qui ad H æquales, vel communes) :: [d] A C. C L. (ob similia trigona C A F. C L H); nam & hìc ang. A C F [a] = ang. L C H. & anguli F A C, C L H [c] recti sunt). ergo permutando D L. C L :: (B D. A C [f] :: B K. A K [g] :: B M. A M [f] ::) D E. E C. ergo invers̀e C L. D L :: E C. D E. & divis̀e C D. D L :: C D. D E. quare D L [h] = D E. [i] *Q. E. A.* Ergo H E potiùs est perpendicularis ad E C. *Q. E. D.*

a 46. *hujus.*
b 15. 1.
c *hyp.*
d 4. 6.
e 45. *hujus.*
f 4. 6.
g 36. 1. *huj.*
h 2. & 4. 6.
i 9. 5.
l 19. 22. 1.

Prop.

Not

P

sit eni

ergo P

PM×MH+

Not: 2da

SX×XL

sit enim

ergo S

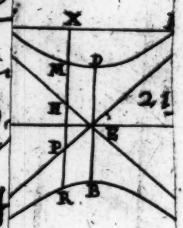
LK×KS-

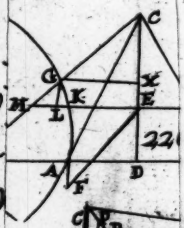
Not: 3t

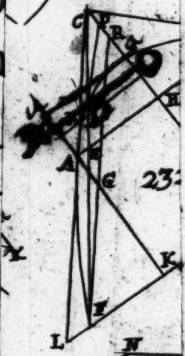
sit enim

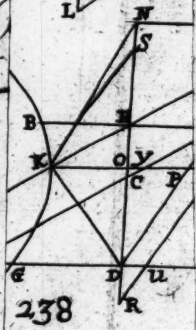
ergo O

TX×XK-









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Not: 1ma $PX \times XH - PM \times MH = RX \times XM$ quia
$PX \times XH = PM \times MH + RX \times XM$
fit enim $AP = b$. fit MH vel $PR = c$. fit $MX = d$
ergo $PX \times XH = cb + cc + 2d + db + dd$. et
$PM \times MH + RX \times XM = cb + cc + 2dc + db + dd$

Not: 2da $SX \times XL - LK \times KS = TX \times XK$ quia
$SX \times XL = LK \times KS + TX \times XK$
fit enim ST, vel $KL = b$. fit $SX = c$. fit $XK = d$
ergo $SX \times XL = bb + bc + bd + dc$ et
$LK \times KS + TX \times XK = bb + bc + bd + dd$.

Not: 3tia $OX \times XN = TX \times XK + OT \times TN$
fit enim OT vel $KN = b$. fit $TX = c$. fit $XK = d$.
ergo $OX \times XL = bb + bc + bd + cd$ et
$TX \times XK + OT \times TN = cd + bb + bc + bd$.

Apollonius. pag. 82.

Prop. XLVIII.

Iiſdem poſitis, oſtendendum eſt lineas (E F, E G), quæ à tactu ducuntur ad puncta (F, G) ex comparatione facta, æquales continere angulos (C E F, G E D) ad contingentem (C D). Fig. 253. 254.

Circulus enim diametro D H deſcriptus [a] tranſit per puncta E, G, [b](ob angulos D G H, D E H rectos) unde ang. D E G [c]=(ang D H G [d]=ang. C H F [c]=) ang. C E F.

Not. In hyperbole anguli D H G, C H F ſunt idem angulus.

[a] 31. 3.
[b] 45. et 47. *h.*
[c] 21. 3.
[d] 15. 1.

Prop. XLIX.

Iiſdem poſitis, ſi à punctorum aliquo (G) ad contingentem (C D) agatur perpendicularis (G H); quæ à facto puncto (H) ducuntur ad axis extrema (A, B) angulos rectos (A H B) continebunt. Fig. 255. 256.

Nam (ob angulos D B G, D H G [a] rectos) circulus ſuper diametro D G deſcriptus [b] tranſit per puncta B, H. unde ang. G H B [c] = ang. B D G [d]= ang A G C [e]= ang A H C [f]= ang G H B. ergo (addito communi angulo B H C, vel A H G) [g] erit ang. A H B=ang. C H G [h]= rect. *Q. E. D.*

Not. ang. A G C=ang. A H C. Nam (ob [h]rectos angulos C A G, C H G) circulus, diametro C G deſcriptus, [b]per puncta A, H tranſibit, in quo anguli A G C, A H C eidem arcui A C inſiſtent, & proinde [c]æquales erunt.

[a] *hyp.*
[b] 31. 3.
[c] 21. 3.
[d] 45. *hujus.* (or 47 *h.*)
[e] *Not.*
[f] 1. *ax.* 1.
[g] 2 *ax.* 1.
[h] *hyp.*

Prop. L.

Iiſdem poſitis, ſi à ſectionis centro (H) ducatur linea (H L) contingenti occurrens, æquidiſtánſque lineæ (F E) per tactum (E) & per unum (F) punctorum ductæ; erit (H L) æqualis dimidio (H B) axis (A B). Fig. 257. 258.

Jungantur E G, A L, L G, L B, ducatúrque G M ad E F parallela. Eſtque (ob F H [a]= H G) E N [b]= N G; ideóq; E L [c] = L M. Item (ob ang. D E G [d]=(ang C E F [e]=) ang E M G.) [f] Erit E G = G M. quare ang. G L E [g]= ang. G L M. ergo G L ad E M eſt perpendicularis. [h] ergo ang A L B eſt rectus. [i] ergo circulus diametro A B deſcriptus tranſibit per L, & radius H L radio H A æquabitur. *Q. E. D.*

[a] *hyp.*
[b] 2. 6.
[c] 48. *hujus.*
[d] 29. 1.
[e] 6. 1.
[f] 8. 1.
[g] 49. *hujus.*
[h] 31. 3.

Prop. LI.

Fig. 259. Si in hyperbola, vel oppoſitis ſectionibus ad axem (A B) comparetur rectangulum (A D B, A E B) æquale quartæ parti figuræ, excedénſque figura quadrata; & à punctis (D, E) ex comparatione factis ad quamlibet ſectionem inclinentur rectæ lineæ (D F, E F); major (E F) minorem (D F) quantitate axis (A B) ſuperabit.

a 29. 1.
b 48. *hujus.*
c 6. 1.
d 2. 6.
e 4. 6.
f 50. *hujus.*
g 1. *ax.* 1.

Recta F K H tangat ſectionem in F, & per centrum C ducatur G C H ad F D parallela. Eſtque ang. K H G [a] = (ang K F D [b] =) ang G F H. unde G H [c] = (G F [d] =) G E (ob C D = C E). Item F D [e] = 2 G C. & C H [f] = C B (ideóque 2 C H = A B). [g] ergo E F (hoc eſt 2 G H) = (2 G C + 2 C H =) F D + A B. *Q. E. D.*

Prop. LII.

Fig. 260. Si in ellipſi ad majorem axem (A B) ex utraque parte comparetur rectangulum (A C B, A D B) æquale quartæ parti figuræ, deficiénſq; figurâ quadratâ; & à punctis (C, D) ex comparatione factis ad ſectionem inclinentur rectæ lineæ (C E, D E), ipſi axi (A B) æquales erunt.

a 48. *hujus.*
b 29. 1.
c 6. 1.
d 2. 6.
e *hyp.*
f 1. *ax.* 1.
g 4.
h 50. *hujus.*

Recta F E H tangat ſectionem, & per centrum G ducatur G K H ad C E parallela. Eſtque ang H E K [a] = (ang F E C [b] =) ang E H G. unde K H [c] = (K E [d] =) K D (ob G C [e] = G D). [f] ergo C E [b] (2 G K) + E D (2 K H) = 2 G H. [g] = 2 A G [e] = A B [f] = C E + E D.

Prop. LIII.

Fig. 261. 262. Si in hyperbola, vel ellipſi, vel circuli circumferentia, vel oppoſitis ſectionibus ab extremo diametri (A C) ducantur lineæ (A D, C E) ordinatim applicatis æquidiſtantes; & à dictis terminis (A, C) ad idem ſectionis punctum (B) ductæ lineæ (A B, C B) ſecent æquidiſtantes (A D, C E); rectangulum ex abſciſſis (A D, C E) factum, æquale erit figuræ, quæ ad eandem diametrum (A C) conſtituitur.

a 4. 6.
b 23. 6.
c 21. 1. *huj.*
d 1. 6.
e 11. 5.
f 9. 5.

Ordinatim applicetur B F. Eſtque A F. F B [a] (hoc eſt A C. C E) + C F. F B [a] (hoc eſt A C. A D) [b] = A F * C F. F B q [c] :: T R. [d] :: Tq. (A C q). T R [e] = A C. C E + A C. A D [b] = A C q. C E * A D. [f] ergo C E * A D = T R. *Q. E. D.*

Prop.

Prop. LIV.

Si coni ſectionem, vel circuli circumferentiam contingentes duæ rectæ lineæ (A D, C D) ſibi ipſis occurrant; & per tactus (C, A) ducantur contingentibus æquidiſtantes (C G, A F); à tactibus verò ad idem ſectionis punctum (H) ductæ lineæ (A H, C H) æquidiſtantes ſecent (in G, & F); rectangulum conſtans ex abſciſſis (A F, C G) ad quadratum lineæ (A C) tactus conjungentis, proportionem habebit compoſitam ex proportione, quam habet quadratum portionis (E B) lineæ (D E) ab occurſu (D) contingentium ad punctum (E) medium conjungentis tactus ductæ, quæ eſt intra ſectionem ad quadratum reliquæ (B D), & ex proportione, quam habet rectangulum ex contingentibus (A D, C D) factum, ad quartam partem quadrati lineæ (A C) tactus conjungentis. Fig. 263.

Per H, B ducantur K H O X L, M B N ad A C parallelæ. Liquet M N ſectionem [a] tangere, & fore M B [b] = B N & K O [b] = O L. (nam A E [c] = E C) item H O [d] = O X; [e] quare H K = X L. & [f] X K = L H. Hinc A Mq. M B × B N (M Bq) [g] :: A Kq. X K × K H (L H × K H). & permutando A Mq. A Kq :: M B × B N. L H × K H. [h] item N C × A M. A Mq :: L C × A K. A Kq. ergo ex æquali N C × A M. M B × B N :: L C × A K. L H × K H [i] = L C. L H [m] (F A. A C) + A K. K H. [m] (G C. A C) [l] = F A × G C. A Cq [n] :: N C × A M. M B × B N [o] = N C × A M. N D × D M [p] (E Bq. B Dq) + N D × D M. M B × B N [q] (C D × D A. A E × E C) [n] ergo F A × G C. A Cq = E Bq. B Dq + C D × D A. A E × E C. *Q. E. D.*

a 32. 1.
b *ex.* 4. 6.
c *hyp.*
d 46, & 47 1.
e *hujus.*
f 3. *ax.* 1.
g 2. *ax.* 1.
h 16. *hujus.*
i *Not.* 1.
l 23. 6.
m 4. 6.
n 11. 5.
o 5. *def.* 6.
p *Not.* 2.
q *Not.* 3.

Not. 1. N C × A M. A Mq :: L C × A K. A Kq. Nam (propter parallelas A C, K L, M N) eſt A M. A K [a] :: N C. L C. & permutando A M. N C :: A K. L C. [b] hoc eſt A Mq. A M × N C :: A Kq. A K × L C. & inverſè.

a 2. 6.
b 1. 6.

Not. 2. N C × A M. N D × D M :: E Bq. B Dq. Nam A M × C N. N D × D M [c] = (A M. D M [d] (E B. B D) + C N. N D [d] (E B. B D) = E B. B D + E B. B D [d] =) E Bq. B Dq.

c 23. 6.
d 2. 6.

Not. 3. N D × D M. M B × B N :: C D × D A. A E × E C. Nam N D × D M. M B × B N [e] = (N D. B N [f] (hoc eſt C D. E C) + D M. M B [f] (hoc eſt D A. A E) = C D. E C + D A. A E [e] =) C D × D A. E C × A E.

e 23. 6.
f 4. 6.

Prop.

Prop. LV.

Fig. 264. Si quæ oppoſitas ſectiones contingunt, rectæ lineæ (A G, D G) ſibi ipſis occurrant; & per occurſum (G) ducatur linea (C E) conjungenti tactus (A D) æquidiſtans; per tactus verò ducantur contingentibus æquidiſtantes (A M, D M); & à tactibus ad idem alterius ſectionis punctum (F) ducantur lineæ (A F, F D), quæ ſecent æquidiſtantes (D M, A M), rectangulum conſtans ex abſciſſis (A H, D N) ad quadratum lineæ (A D) tactus conjungentis, eandem proportionem habebit, quam rectangulum ex contingentibus (A G, D G) factum ad quadratum lineæ (C G) ab occurſu (G) ad ſectionem ducta, quæ quidem (C G) conjungenti tactus (D A) æquidiſtet.

Per F ducatur F L K B ad A D parallela. Eſtque E Gq. (vel C Gq) G Dq [a] :: B L × L F (vel K F × L F). L Dq. item [b] G Dq. G D × G A :: L Dq. L D × A K. ex æquali igitur C Gq. G D × G A :: K F × L F. L D × A K [c] = K F. A K [d] (A D. D N) + L F. L D [d] (A D. A H) = A D. D N + A D. A H [d] = A Dq. D N × A H. ac inverſè G D × G A. C Gq :: D N × A H. A Dq. Q. E. D.

[a] Not. 1. [a] 20. hujus. [b] Not. 2. [c] 23. 6. [d] 4. 6.

Not. 1. E G = C G. & L F = K B, vel K F = L B. Nam bifectâ A D in O, [e] erit G O recta diameter, cui conjugata quæ ad A D parallela. ergo C G = G E. & B P = P F. [f] item K P = P L. [g] ergo K B = L F &c.

[e] 38. 2 hujus. [f] ex. 4. 6. [g] 3. ax. 1.

Not. 2. G Dq. G D × G A :: L Dq. L D × A K. Nam propter parallelas A D, K L, erit G D. L D [h] :: G A. K A. & permutando G D. G A. [k] (hoc eſt G Dq. G D × G A) :: L D. K A [k] (hoc eſt) :: L Dq. L D × K A.

[h] 4. 6. & converſè. [k] 1. 6.

Prop. LVI.

Fig. 265. Si quæ unam oppoſitarum ſectionum contingunt, rectæ lineæ (A E, B E) ſibi ipſis occurrant; & per tactus (A, B) ducantur contingentibus æquidiſtantes (B N, A M); à tactibus verò ad idem alterius ſectionis punctum (C) ducantur lineæ (A C, B C), quæ æquidiſtantes (B N, A M) ſecent (in N, M): rectangulum conſtans ex abſciſſis (B N, A M) ad quadratum lineæ (A B) tactus conjungentis proportionem habebit compoſitam ex proportione, quam habet quadratum portionis (L D) lineæ, ad punctum L medium conjungentis tactus (A B) ductæ, quæ eſt inter dictum punctum (L), & alteram ſectionem (D) ad qua-

quadratum ejus (DE), quæ inter ſectionem (D), & occurſum (E) interjicitur; & ex proportione, quam habet rectangulum, ex contingentibus (AE, BE) factum ad quartam partem quadrati lineæ (AB) tactus conjungentis.

Ducantur CGK, DHF ad AB parallelæ. Eſtque BHq. HDq [a](HD×DF) [b]::BKq. PK×KC [a](CG×KC). item FA×BH. BHq [c]:: GA×BK. BKq. ergo ex æquali FA×BH. HD×DF :: GA×BK. CG×KC. atqui FA×BH. HD×DF [d]= FA×BH. HE×EF [e](LDq. DEq) + HE×EF. HD×DF [f](AE×EB. AL×LB). [g]ergo LDq. DEq + AE×EB. AL×LB= GA×BK. CG×KC [h]= BK. KC [k](AM. AB) + GA. CG [k](NB. AB) = AM×AB + NB. AB [h]= AM×NB. ABq [g]= LDq. DEq + AE×EB. AL×LB. *Q. E. D.*

a *Not.* 1.
b 18. *hujus.*
c *Not.* 2.
d 5. *def.* 6.
e *Not.* 3.
f *Not.* 4.
g 11. 5.
h 23. 6.
k 4. 6.

Not. 1. HD = DF. & PK = CG. Nam ob AL [l]= LB [m]eſt HD = DF. [m]& KX = XG. item XC [n]= XP. [o]ergo PK = CG.

l *hyp.*
m *ex.* 4. 6.
n 47. 1. *huj.*
o 2. *ax.* 1.

Not. 2. FA×BH. BHq :: GA×BK. BKq. Nam propter parallelas BA, FH, GK, [p]erit FA. AG :: HB. BK. & permutando FA. HB [q](hoc eſt FA×HB. HBq) :: AG. BK (hoc eſt AG×BK. BKq).

p *ex.* 2. *vel* 4.
q 1. 6.

Not. 3. FA×BH. HE×EF :: LDq. DEq. Nam FA×BH. HE×EF [r]= FA. EF [s](LD. DE) + BH. HE [s](LD. DE) = LD. DE + LD. DE [r]= LDq. DEq.

r 23. 6.
s 1. 2. 4. 6.

Not. 4. HE×EF. HD×DF :: AE×EB. AL×LB. Nam HE×EF. HD×DF [t]= HE. HD [u](EB. BL) + EF. DF [u](EA. AL) = EB. BL + EA. AL [t]= EB×EA. BL×AL.

t 23. 6.
u 4. 6.

APOL-

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LIB. IV.

Prop. I.

Fig. 266. SI in coni ſectione, vel circuli circumferentia ſumatur aliquod punctum (D) extrà; atque ab eo ad ſectionem ducantur duæ rectæ lineæ (D B, D C); una quidem (D B) contingens, altera verò (D C) in duobus punctis (E, C) ſecans; & quam proportionem habet tota linea ſecans (C D) ad partem ſui ipſius (D E), quæ extra ſumitur, inter punctum (D) & ſectionem interjecta; in eandem dividatur, quæ (C E) eſt intra, ita ut rectæ lineæ ejuſdem rationis ad unum punctum conveniant (C D. D E :: C F. F E); quæ à tactu (B) ad diviſionem (F) ducitur, occurret ſectioni; & quæ ab occurſu ducitur ad punctum (D) extra ſumptum, ſectionem continget.

Eſt quaſi converſa 37æ 3ii.

a 49. 2. *huj.* b 37. 3. *hujus.* c *hyp.* d 9. 5.

Ex D [a] ducatur tangens D A, & connectatur A B, ſecans D C in G. ergo C G. G E [b] :: (C D. D E [c] ::) C F. F E. ergo componendo C E. G E [d] :: C E. F E. ergo G E = F E. ergo puncta G, F coincidunt. *Q.E.D.*

Coroll. B A ſecat D C in F.

Prop. II.

Hæc quidem communiter in omnibus ſectionibus demonſtrata ſunt; at in ſola hyperbola ſi linea D B ſectionem contingat, & D C in punctis E, C ſecet, puncta verò E, C contineant tactum ad B, & punctum D ſit extra angulum aſymptotis comprehenſum, ſimiliter fiet

intra

de-

demonſtratio. * Poſſumus enim à puncto D aliam ducere contingentem D A, & quæ reliquæ ſunt ad demonſtrationem, perficere. * 49. 2. *huj.*

Prop. III.

Iiſdem exiſtentibus puncta E, C punctum B non contineant, ſitque punctum D intra angulum aſymptotis comprehenſum, poterimus à puncto D alteram contingentem ducere, quæ ſit D A, & reliqua ſimiliter demonſtrare. Fig. 267.

Prop. IV.

Iiſdem poſitis, ſi occurſus E, C contineant tactum ad B; & punctum D ſit in angulo (L X N), qui deinceps eſt angulo (K X N) aſymptotis (K L, M N) comprehenſo; linea quæ à tactu (B) ad diviſionem (F) ducitur, occurret oppoſitæ ſectioni; & quæ ab occurſu ducitur eandem ſectionem continget. Fig. 268.

Ex D ducatur D H tangens oppoſitam ſectionem, & connectatur H B, ſecans C E in G. [a] ergo C G. G E :: (C D. D E [b] ::) C F. F E. & componendo C E. G E :: C E. F E. [c] ergo puncta G, F coincidunt. Q. E. D. a 49. 2. *huj.* b 37. 3. *huj.* c *hyp.* d 9. 5.

Coroll. H B ſecat D C in F.

Prop. V.

Iiſdem poſitis, ſi punctum D ſit in una (X N) aſymptoton; quæ à puncto B ad F ducitur, eidem aſymptoto (X N) æquidiſtabit. Fig. 269.

Nam per B ducta B G ad X N parallela, erit rurſus C G. G E [a] :: (C D. D E [b] ::) C F. F E. ergo componendo, C E. G E :: C E. F E. [c] unde G, & F coincidunt. a 35. 3. *huj.* b *hyp.* c 9. 5.

Prop. VI.

Si in hyperbola extra ſumatur aliquod punctum (D), à quo ad ſectionem ducantur duæ rectæ lineæ (D B, D F); altera quidem (D B) contingens, altera vero (D F) æquidiſtans uni aſymptototon; & æquidiſtantis portio (E D) inter ſectionem & punctum (D) interjecta, æqualis ſit ei (E F), quæ intra ſectionem continetur: linea, quæ à tactu (B) ducitur ad factum punctum (F), occurret ſectioni; & quæ ab occurſu ducitur ad punctum extra ſumptum (D) ſectionem continget. Fig. 270.

Ponatur D intra angulum asymptotis comprehensum; ex D [a] ducatur tangens D A, conjungatur B A, secans D F in G. ergo E G [b] = E D [c] = E F, quare puncta E, F coincidunt. *Q. E. D.*

Coroll. A B secat D E in F.

a 49. 2. *hujus.*
b 30. 3. *hujus.*
c *hyp.*

Prop. VII.

Fig. 271. Iisdem positis, sit punctum D in angulo deinceps ei, qui asymptotis continetur. Dico etiam sic eadem evenire.

Iterum, ex D ducatur tangens D H, & connectatur H B secans D F in G. ergo E G [a] = E D [b] = E F. ergo non differunt puncta F, G. *Q. E. D.*

a 31. 3. *huj.*
b *hypoth.*

Prop. VIII.

Fig. 272. Iisdem positis, sit punctum D in asymptoton unâ (M N), & reliqua eadem fiant: dico lineam, quæ à tactu (B) ad extremam partem (F) sumptæ (D F) ducitur, æquidistantem esse asymptoto (M N), in qua est punctum D.

Nam ducatur B G ad M N parallela, ergo rursus E G [a] = (E D) [b] = E F. & puncta G, F coincidunt. *Q. E. D.*

a 34. 3. *huj.*
b *hyp.*

Prop. IX.

Fig. 273. Si ab eodem puncto (D) ducantur duæ rectæ lineæ (D E, D F), quarum utraque coni sectionem, vel circuli circumferentiam in duobus punctis secet; & quam proportionem habent totæ lineæ (E D, F D) ad portiones (D H, D G); quæ extra sumuntur, in eam dividantur, quæ sunt intra (E H, F G); ita ut partes ejudem rationis ad idem punctum conveniant (E D. D H :: E K. K H. & F D, D G :: F L. L G), quæ per divisiones (L, K) ducitur linea, sectioni in duobus punctis occurret; & quam ab occursu ad punctum (D) extra sumptum ducuntur, sectionem contingent.

A puncto D [a] ducantur contingentes D A, D B; & conjungatur A B. hæc [b] secat D E in K; [b] & D F in L. unde liquet propositum.

a 49. 2. *huj.*
b *cor.* 1. 4. *h.*

Prop.

Prop. X.

Hæc quidem communiter in omnibus, at in ſola hyperbola, ſi aliæ quidem eadem ſint, unius autem rectæ lineæ occurſus contineant occurſus alterius; & punctum D ſit intra angulum aſymptotis comprehenſum, eadem prorſus evenient, quæ dicta ſunt, ut in ſecundo theoremate tradidimus.

Prop. XI.

Iiſdem poſitis, ſi unius lineæ occurſus occurſus alterius non contineant, & punctum D ſit intra angulum aſymptotis comprehenſum, & figura, & demonſtratio eadem erit, quæ in tertio theoremate.

Prop. XII.

Iiſdem poſitis, ſi unius lineæ (D F) occurſus (F, G) alterius (D E) occurſus (E, H) contineant; & ſumptum punctum (D) ſit in angulo (P R X) deinceps ei, qui aſymptotis (P O, N X) comprehenditur; linea per diviſiones (K L) ducta, ſi producatur, occurret oppoſitæ ſectioni; & quæ ab occurſibus ducuntur ad punctum D, oppoſitas ſectiones contingent. Fig. 274.

Ex D ducantur contingentes D M, D S; & connectatur M S: hæc [a] ſecat ipſam D E in K, ipſamque D F in L. unde liquet propoſitum. [a] *cor. 4. 4. huj.*

Prop. XIII.

Iiſdem poſitis, ſi punctum D ſit in una aſymptoton, & reliqua eadem exiſtant, quæ per diviſiones (K L) tranſit linea, aſymptoto (O P) in qua eſt punctum, æquidiſtabit; & producta occurret ſectioni; quæ vero ab occurſu ad punctum (D) ducitur, ſectionem continget. Fig. 275.

Itidem liquet ex 5[a] hujus.

Prop. XIV.

Iiſdem poſitis, ſi punctum D ſit in una (P O) aſymptoton; & linea quidem D E ſectionem in duobus punctis ſecet; D G verò alteri aſymptoto (N X) æquidiſtans, ſecet in uno tantùm, quod ſit G; fiátq; ut E D ad D H, ita E K ad K H; & ipſi D G ponatur æqualis G L; quæ per puncta K, L tranſit linea & aſymptoto (O P) æquidiſtabit, Fig. 276.

& ſectioni occurret: quæ verò ab occurſu (B) ducitur ad D, ſectionem continget.

Nam ductâ contingente D B, & per tactum B ductâ ad aſymptoton O P parallelâ; ſecabit hæc [a] ipſam D E in K, [b] ipſamque D G in L, unde liquet propoſitum.

a cor. 1. 4. huj.
b cor. 6. 4. huj.

Prop. XV.

Fig. 277. Si in ſectionibus oppoſitis (A, B) inter duas ſectiones ſumatur aliquod punctum (D), & ab ipſo duæ lineæ ducantur; altera quidem (DF) contingens unam oppoſitarum, altera verò (A B) utramque ſecans; & quam proportionem habet linea (A D) inter ſectionem (A) quam non contingit, & punctum (D) interjecta ad lineam (D B), quæ eſt inter punctum & alteram ſectionem (B), eandem habaet linea quædam (A C) major eâ (A B), quæ inter ſectiones interjicitur, ad exceſſum ipſius (C B) in eadem recta, & ad eundem terminum (B) cum linea ejuſdem rationis; quæ a termino (C) majoris lineæ (A C) ad tactum (F) ducitur, occurret ſectioni, & quæ ab occurſu ducitur ad ſumptum punctum (D), ſectionem continget.

Ex D [a] ducatur tangens altera D E; & connectatur F E. Hæc ſecat A C in C; ſi negas, ſecet alibi in G. ergo erit A G. G B [b] :: (A D. D B [c] ::) A C. C B. & dividendo A B. G B :: A B. C B. [d] unde G B = C B. ergo G, & C ſunt idem punctum.

a 49. 2 huj.
b 36. 1 huj.
c hypoth.
d 7. 5.

Prop. XVI.

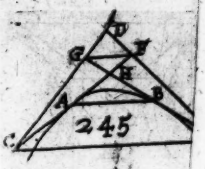
Fig. 278. Iiſdem poſitis ſit punctum D in angulo deinceps ei, qui aſymptotis continetur, & reliqua eadem fiant: dico lineam a puncto F ad C productam occurrere oppoſitæ ſectioni, & quæ ab occurſu ducitur ad D, eandem ſectionem contingere.

Ducatur ex D tangens altera D E; & connectatur E F, ſecans A B in G. ergo A G. G B [b] :: (A D. D B [c] ::) A C. C B. unde dividendo A B. G B :: A B. C B. ergo G B [d] = C B. ergo puncta C, G coincidunt. *Q. E. D.*

a 49. 2 huj.
b 39. 3. huj.
c hyp.
d 7. 5.

Prop. XVII.

Fig. 279. Iiſdem poſitis ſit punctum D in una aſymptoton: dico lineam, quæ ab F ad C ducitur, aſymptoto, in qua eſt punctum (D) æquidiſtare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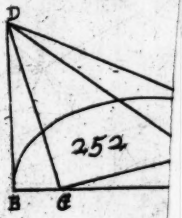
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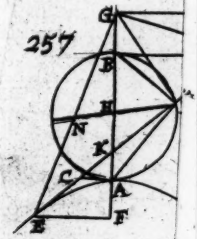
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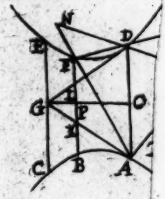

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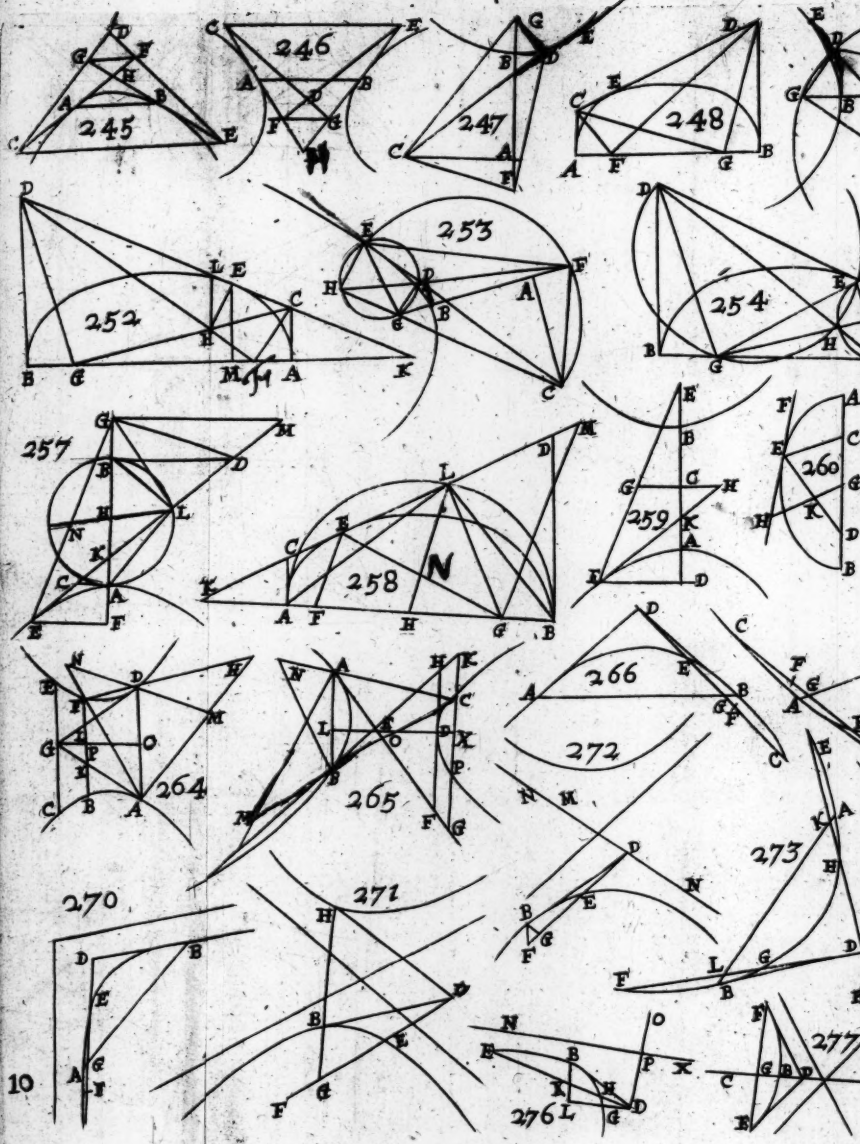
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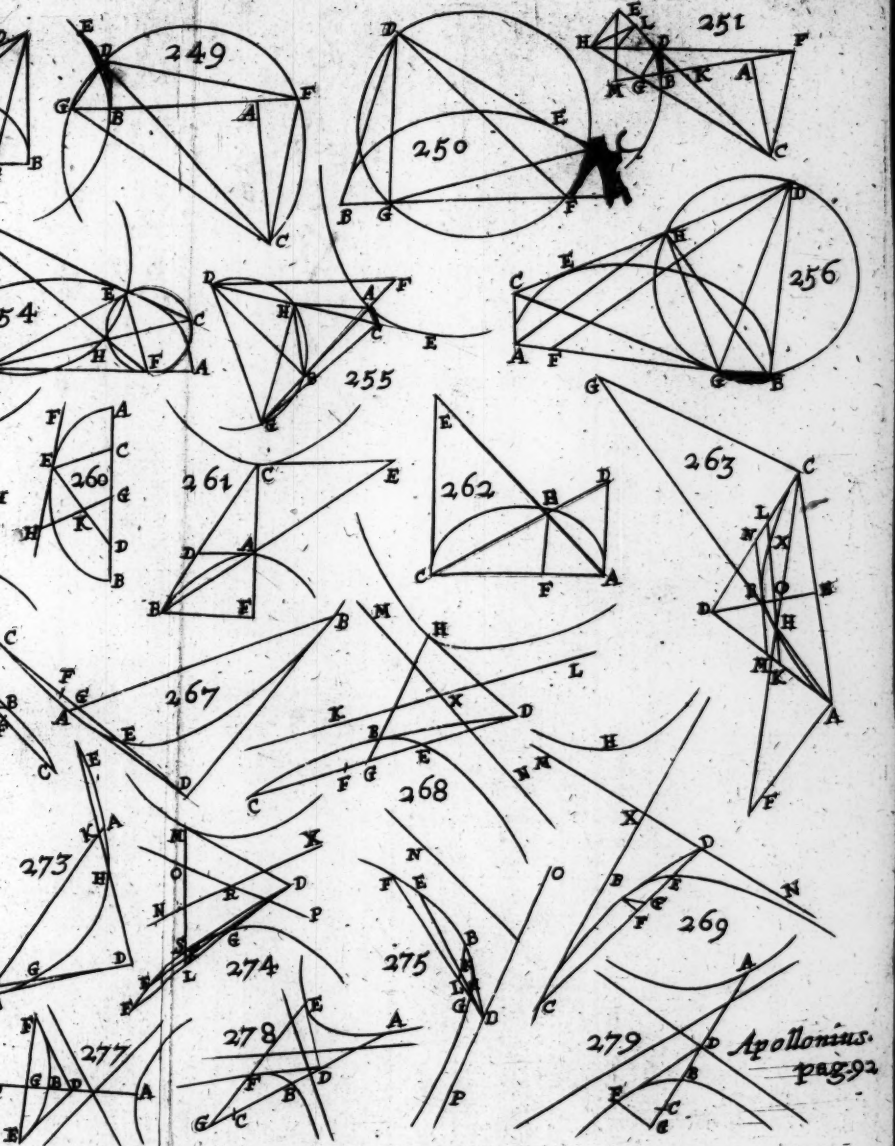
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Apollonius.
pag. 92

Sit F G aſymptoto parallela. ergo rurſus A G. G B [a]:: (A D. D B [b]::) A C. C B. & dividendo A B. G B:: A B. C B. ergo G, & C ſunt idem punctum. [c] Q. E. D.

a 36. 3 *huj.*
b *hyp.*
c *ut in prac.*

Prop. XVIII.

Si in ſectionibus oppoſitis ſumatur aliquod punctum (D) inter duas ſectiones, & ab ipſo ducantur duæ lineæ (A B, C H), utramque ſectionem ſecantes; & quam proportionem habent interjectæ (A D, C D) inter unam ſectionem, & punctum (D) ad eas, (D B, D H), quæ inter idem punctum (D) & alteram ſectionem interjiciuntur, eandem habent lineæ (A K, C G) majores iis (A B, C H), quæ ſunt inter ſectiones oppoſitas, ad exceſſus ipſarum (K B, G H); quæ per terminos (K, G) majorum linearum tranſeunt, occurrent ſectionibus; & quæ ab occurſibus ad ſumptum punctum (D) ducuntur, ſectiones contingent. Ponatur D inter aſymptotos. Fig. 280.

Per D [a] ducantur contingentes D E, D F, & connectatur E F; ſecat hæc [b] ipſam A B in K, [b] ipſamque C H in G. ergo liquet propoſitum.

a 49. 2 *huj.*
b 15. 4. *huj.*

Prop. XIX.

Sumatur punctum D in angulo deinceps ei, qui aſymptotis continetur, ducanturque rectæ lineæ (A B, C H) ſectiones ſecantes, & ut dictum eſt dividantur (in K, G): dico eam, quæ per K G producitur, occurrere utrique ſectionum, & quæ ab occurſibus ducuntur ad D, ſectiones contingere. Fig. 281.

Rurſus enim per D [a] ducantur contingentes D E, D F; [b] ſecabit juncta F E ipſam A B in K, ipſamque C H in G. unde conſtat propoſitum.

a 49. 2. *huj.*
b 16. 4. *huj.*

Prop. XX.

Si ſumptum punctum (D) ſit in una aſymptoton, & reliqua eadem fiant; linea (K G), quæ tranſit per terminos (K G) exceſſuum, aſymptoto, in qua eſt punctum (D) æquidiſtabit; & quæ a puncto (D) ducitur ad occurſum ſectionis, & lineæ (K G) per terminos tranſeuntis, ſectionem continget. Fig. 282.

Nam pariter [a] ductâ tangente D F, ducta per tactum F ad aſymptoton parallela [b] ſecabit A B in K, & C H in G. ergo res conſtat.

a 49. 2 *huj.*
b 17. *hujus.*

Prop.

Prop. XXI.

Fig. 283. Sint rursus oppositæ sectiones A, B, sitque punctum D in una asymptoton, & linea quidem D B K in uno tantùm puncto occurrat sectioni B, alteri asymptoto æquidistans, linea verò C D H G utrique sectioni occurrat; & ut C D ad D H, ita C G ad G H, & ipsi D B æqualis sit B K. Dico lineam, quæ per puncta K, G transit, occurrere sectioni, asymptotóque, in qua & punctum D, æquidistare; & quæ ab occursu ad punctum D ducitur, sectionem contingere.

a 8. 4. *hujus.* b 17. *hujus.* Ductâ enim tangente D F, & F G parallelâ asymptoto (in qua D); secabit F G [a] ipsam D B in K, [b] ipsamque C H in G. unde constat.

Prop. XXII.

Fig. 284. Sint similiter oppositæ sectiones, asymptotique, & punctum D sumatur in angulo deinceps ei, qui asymptotis continetur; linea verò C D H secet utrasque sectiones, & D B alteri asymptoto æquidistat; sitque ut C D ad D H, ita C G ad G H, & ipsi D B æqualis ponatur B K: dico lineam quæ per puncta K, G transit, occurrere utrique oppositarum sectionum; & quæ ab occursibus ducuntur ad D sectiones easdem contingere.

Itidem patet ut in 6ta, & 16ta hujus libri.

Prop. XXIII.

Fig. 285. Sint itidem oppositæ sectiones A B; punctúmque D sit in angulo deinceps ei, qui asymptotis continetur; & linea quidem B D sectionem B in uno puncto tantùm secet, alteri asymptoto æquidistans; linea verò D A similiter secet sectionem A, sitque D B ipsi B G æqualis, & D A ipsi A K; dico lineam, quæ transit per K G occurrere sectionibus; & quæ ab occursibus ad D ducuntur, sectiones contingere.

Patet ut in [illegible]. *Sapbina*

Prop. XXIV.

Fig. 286. Coni sectio (D A B C) coni sectioni vel circuli circumferentiæ (E A B C) non occurrit, ita ut pars quidem eadem sit, pars verò non communis.

Si

Si affirmas, in parte communi ABC ſumatur punctum H utcunque, & connectatur AH; & huic parallela ducatur DC; ipſamque AH biſecet diameter BGF. Itaque (ob ſectionem EBC) [a] erit EF = FC. & (propter ſectionem DBC) DF [a] = FC. [b] proinde EF = DF. *Q. E. A.*

[a] 46. & 47. 1.
[b] 1 ax. (huj.

Prop. XXV.

Coni ſectio coni ſectionem, vel circuli circumferentiam in pluribus punctis, quàm quatuor non ſecat. Fig. 287.

Si fieri poteſt, ſecet quinque punctis A, B, C, D, E proximis: conjungantur rectæ AB, DC, quæ primò occurrant, (ut ſemper [a] fit in parabola & hyperbola) in L. [b] fiátque AL. LB :: AO. OB. [b] & DL. LC :: DP. PC. jungatúrque OP, ſectionibus occurrens punctis R, H. [c] ergo ductæ LR, LH ſectiones contingent: ducatur EL, ſectionibus occurrens in M, G. eritque EN. NM (d :: EL. LM) [e] ⌐ (EL. LG [d] ::) EN. NG. [f] unde NM = NG. *Q. E. A.*

[a] 24. & 25. 2. *hujus.*
[b] 10. 6.
[c] 9. 4 *hujus.*
[d] 37. 3 *b.*
[e] 8. 9.
[f] 10. 5.

Sin parallelæ ſint AB, DC (ut fieri poteſt in ellipſi & circulo), biſecentur ipſæ in O, & P; & connexa OP ſectionibus occurrat in R, H. [h] quare RH eſt diameter, ad quam ordinatim applicantur AB, DC; eíſque parallela EG (per E ducta). [k] unde erit utraque EG, EM biſecta in N. ac propterea NG = NM. *Q. E. A.*

[g] 9. ax. Fig. 288.
[h] 28. 1. *huj.*
[k] 10. *def.* 1. *h.*
[l] 9. ax.

Prop. XXVI.

Si dictarum linearum aliquæ in uno puncto (A) ſeſe contingant, non occurrant ſibi ipſis ad alia puncta plura quàm duo. Fig. 289.

Si fieri poteſt, occurrant tribus punctis B, C, D proximè ſitis; [a] ducatúrque tangens AL, occurrens ductæ BC in L; [b] fiat autem BL. CL :: BP. PC; & connexa AP ſectionibus occurrat in H, & R. [c] ergo ductæ LH, LR contingent ſectiones: itaque ducta DL, abſurditatem incurremus, pariter ac in præcedenti. Sin BC, AL non conveniant, (in ellipſi nempe, & circulo) ſimiliter conſequetur abſurdum.

[a] 49. 2. *huj.*
[b] 10. 6.
[c] 9 *hujus.*

Prop. XXVII.

Si prædictarum linearum aliquæ in duobus punctis (A, B) ſeſe contingant, in alio puncto ſibi ipſis non occurrent. Fig. 290.

Occurrant, ſi fieri poteſt, in C; primò extra contactus. 291.
292.

Du-

Ducantur tangentes A L, B L, quæ si occurrant in L, ductâ C L, incurritur absurditas, ut in 25ma. Sin tangentes parallelæ sint, [a] erit quoque A B utriusque sectionis diameter, ac [b] idcirco N M [b] = (N C [b] =) N G. [c] *Q. E. A.*

a 27. 2 *hujus.*
b 10 *def.* 1. *b.*
c 9. *ax.*

Sin punctum C sit intra contactus, rursus ducantur tangentes A L, B L, & connexa A B bisecetur in F, [d] eritque F L diameter; unde si ducatur G G M ad A B parallela, [b] erit utraque C G, C M bisecta in K; unde K G = K M. [c] *Q. E. A.*

d 29. 2. *huj.*

Prop. XXVIII.

Fig. 293. Parabole (A G B) parabolen (A M B) non contingit, præterquam in uno puncto.

Si fieri potest, contingant se parabolæ punctis (A, B), à quibus ducantur tangentes A L, B L, concurrentes in L. junctámque A B bisecet recta L F. [a] hæc diameter erit utriusque sectionis; [b] unde L F bisecta est in G, & M. [c] *Q. E. A.*

a 29. 2 *huj.*
b 35. 1 *huj.*
c 9. *ax.*

Prop. XXIX.

Fig. 293. Parabole (A G B) hyperbolen (A M B) non contingit in duobus punctis, extra ipsam cadens.

Tangat, si fieri potest, punctis A, B, à quibus ducantur tangentes A L, B L; junctámque A B bisecet recta L F; [a] hæc utriusque sectionis erit diameter. Sit D centrum hyperbolæ; eritque F D. D M [b] :: (D M. D L [c] ::) F M. M L. quare, cùm F D ⊏ D M, [d] erit F M ⊏ M L. ergo F M ⊏ G L. atqui F G [e] = G L. ergo F M ⊏ F G. [f] *Q. E. A.*

a 29. 2 *huj.*
b 37. 1. *huj.*
c 19. 5.
d 14. 5.
e 35. 1. *huj.*
f 9. *ax.*

Prop. XXX.

Fig. 293. Parabola (A M B) ellipsim vel circuli circumferentiam (A G B) non contingit in duobus punctis (A, B) intra ipsam cadens.

Si affirmas, ducantur tangentes A L, B L. & rectam A B iterum bisecet diameter F G, in qua sit D centrum ellipsis, vel circuli. Estque L D. D G [a] :: (D G. D F [b] ::) L G. G F. ergo L G ⊏ G F. atqui F G = G L. quæ [d] repugnant.

a 37. 1 *huj.*
b 19. 5.
c 14. 5.
d 9. *ax.*

Prop.

Prop. XXXI.

Hyperbole (A G B) hyperbolen (A M B), idem centrum (D) habens, in duobus punctis (A, B) non continget. Fig. 293.

Si dicas contingere, ducantur contingentes A L, B L; & juncta D L producatur: [a] bisecabit utique hæc tactus conjungentem A B, in F; [b] estque (propter hyperbolen A G B) D Gq [b] = F D × D L. & (ob hyperbolen A M B) D Mq [b] = F D × D L. [c] quare D Gq = D Mq. [d] Q. E. A.

a 30. 2. *huj.*
b 37. 1. *b.*
c 1. *ax.*
d 9. *ax.*

Prop. XXXII.

Si ellipsis (A M B) ellipsin, vel circuli circumferentiam, (A G B), habens idem centrum (D), in duobus punctis (A, B) contingat, linea (A B) conjungens tactus, per centrum (D) transibit. Fig. 294.

Nam ductis contingentibus A L, B L, siquidem A B per centrum [a] non transit, [b] hæ convenient, puta in L. eritque juncta L D [c] diameter utriusque sectionis; & proinde in una, D Gq [d] = D L × DF; in altera, D Mq [d] = D L × DF. [e] unde D Gq = D Mq. [f] Q. E. A.

a *hyp.*
b 27. 2 *huj.*
c *cor.* 51. 1. *b.*
d 37. 1 *huj.*
e 1. *ax.*
f 9. *ax.*

Prop. XXXIII.

Coni sectio, vel circuli circumferentia (A B C) coni sectioni, vel circuli circumferentiæ (A D B E C), quæ non ad easdem partes conversa habeat, ad plura puncta quàm duo non occurret. Fig. 295. 296.

Si fieri potest, occurrant tribus punctis A, B, C; liquetque rectas A B, C B continere angulum, ad partes in quibus sunt concava lineæ (A B C). paritérque eædem angulum continent ad partes, in quibus concava lineæ A D B E C. ergo lineæ A B C, A D B E C concava habent ad easdem partes, contra hypothesin.

Prop. XXXIV.

Si coni sectio, vel circuli circumferentia (A B F) occurrat uni (A C F) oppositarum sectionum in duobus punctis (A, F); & lineæ (A B F, A C F) quæ inter occursum interjiciuntur ad easdem partes concava habeant; producta linea (A B F) ad occursus, alteri (D) oppositarum sectionum non occurret. Fig. 297.

a 33. 2 huj. Nam recta A F ſectioni D [a] non occurret, ergo nec ſectio A B F.

Prop. XXXV.

Fig. 298. Si coni ſectio, vel circuli circumferentia (A B C) occurrat uni (A) oppoſitarum ſectionum (A, B); non occurret ipſarum reliqua (B) ad plura puncta, quàm duo (B, C).

Nam ut occurrat pluribus, repugnat 33ª hujus.

Prop. XXXVI.

Coni ſectio, vel circuli circumferentia oppoſitis ſectionibus ad plura puncta, quàm quatuor non occurret.

a 35. 4. huj. Etenim ſi uni occurrat, reliquæ ad plura puncta [a] non occurret, quàm duo.

Prop. XXXVII.

Fig. 299. Si coni ſectio, vel circuli circumferentia (C A D) unam oppoſitarum ſectionum (A) concavâ ſui parte contingat, alteri oppoſitarum (B) non occurret.

a 49. 2.
b cor. 32. 1. b.
c 33. 2. b. Per contactum A [a] ducatur recta E F tangens, [b] utramque ſectionem. [c] hæc ſectioni B non occurret, ergo nec linea C A D.

Prop. XXXVIII.

Fig. 300. Si coni ſectio, vel circuli circumferentia (A B C) utramque oppoſitarum ſectionum (A, B) contingat in uno puncto (A & B); oppoſitis ſectionibus in alio puncto non occurret.

a cor. 32. 1. b.
b 33. 2. Ducantur rectæ A D, B E contingentes ſectiones A, B; [a] hæ lineam A B C etiam contingunt. [b] ergo (incluſa his) linea A B C non occurret ſectionibus A, B.

Prop. XXXIX.

Fig. 301. Si hyperbole (A B C) oppoſitarum ſectionum uni (A B D) in duobus punctis occurrat, convexa habens è regione ſita, quæ ipſi (A B C) oppoſitur ſectio (E), oppoſitarum alteri (F) non occurret.

Con-

Conjungatur A B; [a] hæc neutri sectionum E, F occurret. ergo nec ipsæ E, F (quibus illa interjacet) sibi occurrent. a 33. 2 huj.

Prop. XL.

Si hyperbole (A C B) utrique oppositarum sectionum (A, B) occurrat; quæ ipsi opponitur sectio, nulli oppositarum in duobus punctis occurret. Fig. 302.

Si fieri potest, occurrat sectioni A punctis D, E. [a] ergo recta D E neutri sectionum C, B occurret. Quare nec ipsæ sectiones C, B convenient, contra hypothesin. a 33. 2 huj.

Similiter, sectio E non continget utramque A, B. Nam ducta tangens H E [a] neutri C, B occurret; ergo nec ipsæ, itidem contra hypoth.

Prop. XLI.

Si hyperbola (C A B D) utramque oppositarum sectionum (A, B) duobus punctis (C, A; D, B) secet, convexa habens è regione sita; quæ ipsi opponitur sectio (E F) nulli oppositarum (A, B) occurret. Fig. 303.

Sint K G L, M N G asymptoti sectionum A B, E F. liquet rectam C A [a] occurrere asymptoto L G ad partes K, (non ad L); & rectam D B ad partes M, (non ad partes N,) [a] occurrere asymptoto N G, adeóque [b] angulum P H R continere angulum N G L, & propterea sectionem E F. Atqui C A [c] non occurrit sectioni D B O, nec D B sectioni C A X. ergo rectæ C A R, D B P sectionibus C A X, & E F; sectionibúsque D B O, & E F interjacent; & proinde sectio E F neutri C A X, D B O occurret. *Q. E. D.* a 8. 2 huj. b 25. 2 huj. c 33. 2 huj.

Prop. XLII.

Si hyperbole unam oppositarum sectionum in quatuor punctis (A, B, C, D) secet, quæ ipsi opponitur sectio (K), non occurret alteri oppositarum (E). Fig. 304.

Occurrat, si fieri potest, in K; junctæque A B, D C [a] conveniant productæ in L; & [b] fiat A L. L B :: A P. P B. & D L. L C :: D R. R C. [c] ergo connexa P R sectionibus occurret, [c] & quæ ab L ad occursus ducuntur, sectiones contingent; [d] erítque proinde (ob sectionem A F D) ~~[illegible] N K. K L; [d] & (propter sectionem A M D) N F. F L :: N M. M L. [e] quare N K. K L :: N M. M L. Q. E. A.~~ a 25. 2 huj. b 10. 6. c 9. huj. d 36. 1. huj. & 16. 5. e 11. 5.

FN : FL :: NK : KL et propter sectionem AMD
NK : KL :: NM : ML Quare FN : FL :: NM : ML Q. E. A.

Prop. XLIII.

Fig. 305. Si hyperbole (ACB) oppoſitarum ſectionum (AB, C) alteri (AB) in duobus punctis (A, B) occurrat, concava habens ad eaſdem partes; alteri verò (C) occurrat in uno puncto (C); quæ ipſi (ACB) opponitur ſectio (D), nulli oppoſitarum (AB, C) occurret.

Conjungantur (AC, BC; [a] hæ non occurrent ſectioni D; & quoniam ſectioni AB occurrunt, [b] non ſecabunt alibi ſectionem C; ergo ſectionem D continet ſectio C; unde liquet propoſitum.

[a] 33. 2 *huj.*
[b] 34. 4 *huj.*

Prop. XLIV.

Fig. 306. Si hyperbole (AMBC) uni (ABC) oppoſitarum ſectionum (ABC), DEF) occurrat in tribus punctis (A, B, C); quæ ipſi opponitur (DEK) alteri oppoſitarum (DEF) præterquam in uno puncto non occurret.

Si fieri poteſt, occurat punctis D, E; jungantúrque AB, DE; ſintque hæ primò parallelæ, & biſecentur rectâ GH; [a] eritque GH ſectionum diameter; ductâ igitur per C ipſi BA parallelâ CX, productáque HG ad N, [b] erit utraque CX, CO biſecta in N. [c] *Q. E. A.*

[a] 36. 2 *huj.*
[b] 48. 1 *huj.*
[c] 9. *ax.*

Fig. 307. Sin AB, DE conveniant in P, per C ducatur OCR ad AP parallela, & protrahatur DPR. Biſecentur autem AB, DE punctis H, G; per quæ ducantur diametri GS, HS; & rectæ IT, LT, MY ſectiones contingant; [a] unde IT ad DP, & [a] LT ad AP, [a] & MY ad OR (vel AP) erunt parallelæ. quare OR * RC. DR * RE [b] :: (LTq. TIq [b] :: AP * PB. DP * PE [b] :: MYq. YIq [b] ::) XR * RC. DR * RE. [c] unde OR * RC = XR * RC. [d] *Q. E. A.*

[a] 5. 2 *hujus.*
[b] 19. 3. *huj.*
[c] 9. 5.
[d] 9. *ax.*

Prop. XLV.

Fig. 308. Si hyperbole (ABD) unam (D) oppoſitarum ſectionum (ABC, D) contingat, alteram verò (ABC) ſecet in duobus punctis (A, B); quæ ipſi opponitur ſectio (CE) nulli oppoſitarum (ABC, D) occurret.

Occurrant, ſi fieri poteſt, in C; jungatúrque ABF, cui occurrat tangens DF; [a] erit occurſus F intra aſymptotos ſectionis ABD. ergo ducta CF cadit intra angulum BFD. atqui tangens DF [c] non occurrit ſectioni ABC. ergo CF extra ang. BFD cadit, quæ repugnant.

[a] *in* 25. 2 *huj.*
[c] 33. *huj.*

Prop.

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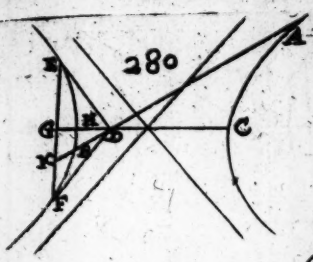
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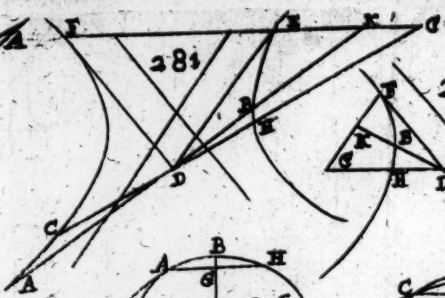
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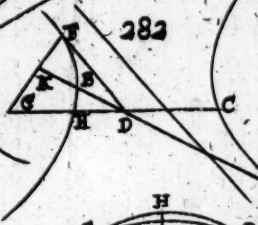
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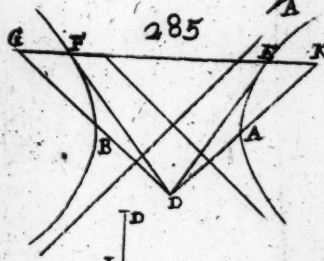
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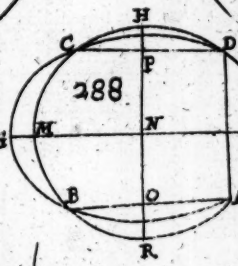
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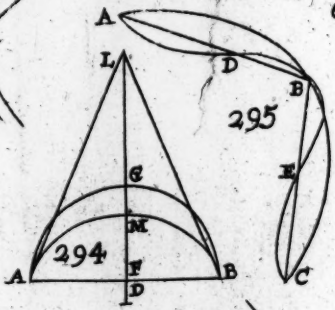
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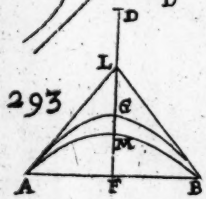
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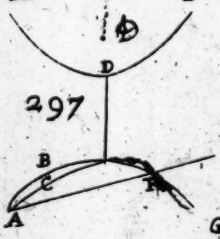
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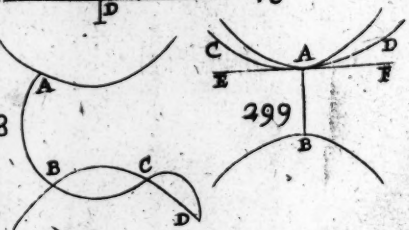
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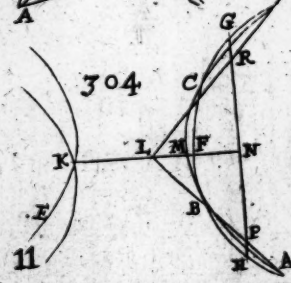
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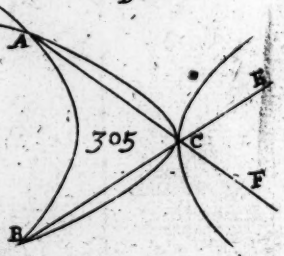
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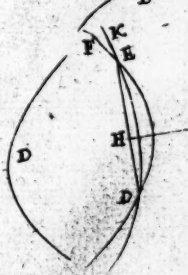

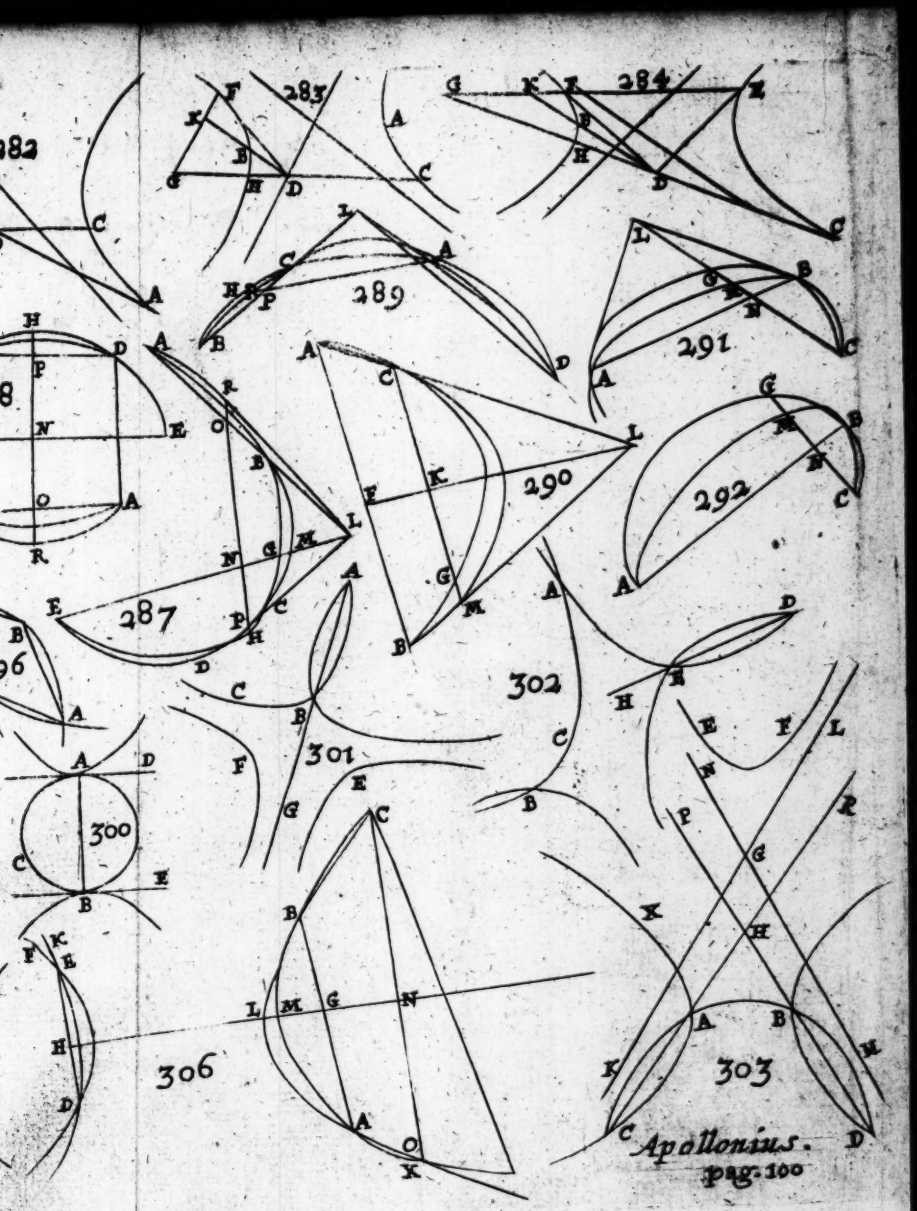
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Prop. XLVI.

Si hyperbole (A G C) unam (A B C) oppoſitarum ſectionum (A B C, D) in uno puncto (A) contingat, & ſecet in duobus punctis (B, C), quæ ipſi opponitur ſectio (E) alteri oppoſitarum (D) non occurret. Fig. 309.

Si fieri poteſt, occurat in D; jungatúrque C B, cui occurrat tangens A F. [a] erit F intra aſymptotos: ducatur D F ſectionem ſecans in G, K. [b] ſitque C F. F B :: C L. L B; & connectatur A L M N. ergo ductæ F M, F N [c] ſectiones contingent; eritque [d] proinde X G. G F :: X D. D F (ob ſectionem A G M); & X K. K F :: X D. D F. (propter ſectionem A K N). [e] quare X G. G F :: X K. K F. *Q. E. A.*

a 25. 2.
b 10. 6.
c 1 *hujus.*
d 36. 1 *hujus.* & 16. 5.
e 11. 5.

Prop. XLVII.

Si hyperbole (D A C) unam (A B C) oppoſitarum ſectionum (A B C, E F G) contingens, in alio puncto (C) ſecet; quæ ipſi D A C opponitur ſectio (E F H) alteri oppoſitarum (E F G) non occurret præterquam in uno puncto. Fig. 310.

Occurrat punctis E, F; jungatúrque E F. Sitque primò E F tangenti A K parallela; ergo quæ biſecat ipſam E F, [a] diameter per A tranſit; ſit hæc A L; perque C ducatur C B ad A K parallela; [b] biſecta eſt igitur utraque B C, D C in L. [c] *Q. E. A.*

a 34. 2 *huj.*
b 48. 1. *huj.*
c 9. *ax.*

Sin E F, A K conveniant (in K); ideóque E F, B C (in N); biſecet diameter A M ipſam E F, ſectiones ſecans punctis X, O; per quæ ducantur contingentes X P, O R, [a] rectæ E F, ac proinde invicem parallelæ; eritque D N * N C. E N * N F [c] :: (A Pq. P Xq [f] :: A Rq. R Oq [c] ::) B N * N C. E N * N F: [g] quare D N * N C = B N * N C. [h] *Q. E. A.*

Fig. 311.
d 34. 2 *huj.*
e 19. 3 *huj.*
f 4. & 22. 6.
g 9. 5.
h 9. *ax.*

Prop. XLVIII.

Si hyperbole (A C) unam (A B) oppoſitarum ſectionum (A B, D E G) in uno puncto (A) contingat; quæ ipſi (A C) opponitur ſectio (D E F) oppoſitarum alteri (D E G) non occurret ad plura puncta, quàm duo. Fig. 312. 313.

Si fieri poteſt, occurrat quoque in H; ducatúrque A K utramque ſectionem A B, A C contingens, & conjungatur D E, quæ primò parallela ſit ipſi A K. Biſecetur D E in L, & connectatur A L: [a] eſt hæc oppoſitarum diameter: ductâque H X G F ad D E parallelâ, erit X G [b] = (X H [b] =) X F. *Q. E. A.*

a 34. 2 *huj.*
b 10 *def* 1 *h.*
c 9. *ax.*

Sin

Sin D E non sit ad A K parallela, occurrat ei in K, reliquisque perfectis ut priùs, producta F H ipsi A K occurrat in R. quare G R × RH. R Aq [d] :: (D K × K E. A Kq ::) F R × R H. R Aq. [c] ideóque G R × R H = F R × R H. [c] *Q. E. A.*

d 19. 3. *huj.*
e 9. 5.

Prop. XLIX.

Fig. 314.

Si hyperbole (A B) utramque oppositarum sectionum (A, B) contingat, quæ ipsi opponitur sectio (E), nulli oppositarum (A, B) occurret.

A C, b C.

Ducantur enim A D, B C, quæ contingant sectiones; [a] liquet has intra asymptotas sectionis A B convenire. [b] ideóque asymptotis sectionis E non occurrere, sed eas continere, & multo magis sectionem E. Cum igitur tangens A C [b] non occurrat sectioni B G, nec tangens B C [b] sectioni A D, neutri harum occurret sectio E. *Q. E. D.*

a 25. 2. *huj.*
b 33. 2. *buj.*

Prop. L.

Fig. 315.

Si utraque oppositarum sectionum (A, D) in uno puncto (A & D) contingant, ad easdem partes concava habens, in alio puncto non occurrent.

Si fieri potest, sectiones D occurrant in E; ergo sectiones A [a] non occurrent præterquam in uno puncto A. ducantur contingentes A H, D H, junctæque A D sit parallela E B C, & per H ducatur H K diameter secunda sectionum oppositarum; [a] bisecabit hæc ipsam A D in K, [c] ideóque utramque E B, E C in L. *Q. E. A.*

a 47. 4. *huj.*
b 39. 2. *huj.*
c 10. *def.* 1. *b.*

Prop. LI.

Fig. 316.

Si hyperbole (A C B) oppositarum sectionum unam (A D B) contingat in duobus punctis (A, B); quæ ipsi opponitur sectio (F) oppositarum alteri (E) non occurret.

Si fieri potest, occurrat in E, ducanturque sectionum contingentes A G, B G; & connectantur A B, E G, & producta E G sectionibus occurrat in C, D, rectæque A B in H. Erit igitur H D. D G [a] :: (H E. E G [a] ::) H C. C G. ergo quum H D ⊐ H C, [b] erit D G ⊐ C G. [c] *Q. E. A.*

a 36. 1. *huj.*
b 14. 5.
c 9 *ax.*

Prop.

Prop. LII.

Si hyperbole (A D) oppositarum sectionum unam (A) contingat (in A), convexa habens è regione sita, quæ ipsi opponitur sectio (F) oppositarum alteri (B) non occurret. Fig. 317.

Ducatur enim A C tangens sectiones; [a] hæc neutri sectionum B, F occurret; sed inter ipsas cadet; ergo nec ipsæ sectiones occurrent. Q. E. D. [a] 33. 2 *hujus.*

Prop. LIII.

Oppositæ sectiones (A B C D, E F) oppositas (A B, C D) non secant in pluribus punctis, quàm quatuor.

Nam quum sectiones convexa habent sibi obversa, si 1°, sectio A B C D * secet utramque A B, C D in quatuor punctis (A, B, C, D), [a] liquet E F neutri reliquarum A B, C D occurrere. *1. Fig. 318. [a] 41. *hujus.*

Sin 2dò, sectio A B C sectionem A B* secet in duobus punctis (A, B,) ipsasque C D in uno C, non occurret ergo E F [b] ipsi C D, nec ipsi A B præterquam uno puncto; (si enim duobus, [c] ergo ei opposita A B C non omnino occurret alteri C D, contra hypoth.) *2. Fig. 319. [b] 39. *hujus.* [c] 41. *hujus.*

Sin 3°, Sectio A B C sectionem A B E secet punctis duobus A, B; [d] non occurret sectio E F sectioni D, nec sectioni A B E, præterquam duobus punctis. 3. Fig. 320. [d] 35. *hujus.*

Sin 4°, Sectio A B C D utramque A B, C F unico puncto secet, [e] nulli ipsarum occurret ipsa E F duobus punctis. 4. Fig. 321. [e] 40 *hujus.*

Sin Sectiones ad easdem partes concava habeant; & 5°, altera alteram in quatuor punctis (A, B, C, D) secet, [f] sectio EF neutri A B, C D occurret. Sin 6°, Sectio A B C alteri occurrat tribus punctis, [g] ipsa E F alteri in uno tantùm puncto occurret, idémque in reliquis. Nullo igitur modo oppositæ sectiones oppositis ad plura puncta convenient, quàm quatuor. 5. Fig. 322. 323. [f] 36. *hujus.* 6. Fig. 324. [g] 44. *hujus.*

Prop. LIV.

Si oppositæ sectiones (B C, E F) oppositas (A B, D) in uno puncto (B) contingant, non occurrent sibi ipsis ad alia puncta plura quàm duo. Fig. 325.

Habeant Sectiones convexa sibi invicem obversa, occurrátque primò Sectio B C ipsi D duobus punctis, C, D. [a] ergo Sectio E F neutri [a] A B, [b] C D occurret. [a] 39. *hujus.* [b] 52 *hujus.*

Sin 2ò, Sectio B C secet ipsam D semel in C, [c] ergo E F sectioni D, nusquam occurret, nec ipsi A B nisi in uno puncto (si enim duobus, [d] non Fig. 326. [c] 52 *hujus.*

d 39 *hujus.* [d] non occurret B C ipſi D) contra hypotheſin).

Fig. 327. Sin 3ò, Sectio B C non occurrat ſectioni D, [e] nec E F ipſi D occurret, [f] ipſique A B occurret duobus tantùm punctis.

e 52 *hujus.*

f 35. *hujus.* Sin Sectiones ad eaſdem partes concava habeant, ſimilis erit diſcurſus; conſtabitque omninò propoſitum.

Prop. LV.

Fig. 328. Si Sectiones oppoſitæ (A B, C D) oppoſitas (A C, E F) contingant in duobus punctis, in alio puncto ſibi ipſis non occurrent.

a 49. *hujus.* Contingant primò in A C. [a] ergo Sectio E F nulli ipſarum occurret.

b 51. *hujus.* Secundò, contingant in A, B; [b] itaque rurſus C D non occurret ipſi E F.

Fig. 329.

Fig. 330. Tertiò, contingat Sectio A C ipſam A B in A, & Sectio D ipſam E F in F. [c] ergo nec Sectio E F Sectioni A B, nec Sectio C A ipſi D F occurret.

c 52. *hujus.*

Fig. 331. Quartò, contingat A C ipſam A B in A, & E F ipſam D in E, habentes concava ad eaſdem partes, liquet ipſas neutiquam in alio puncto occurrere: quare omninò conſtat propoſitum.

d 50 *hujus.*

LAUS DEO.

O
X
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3
H
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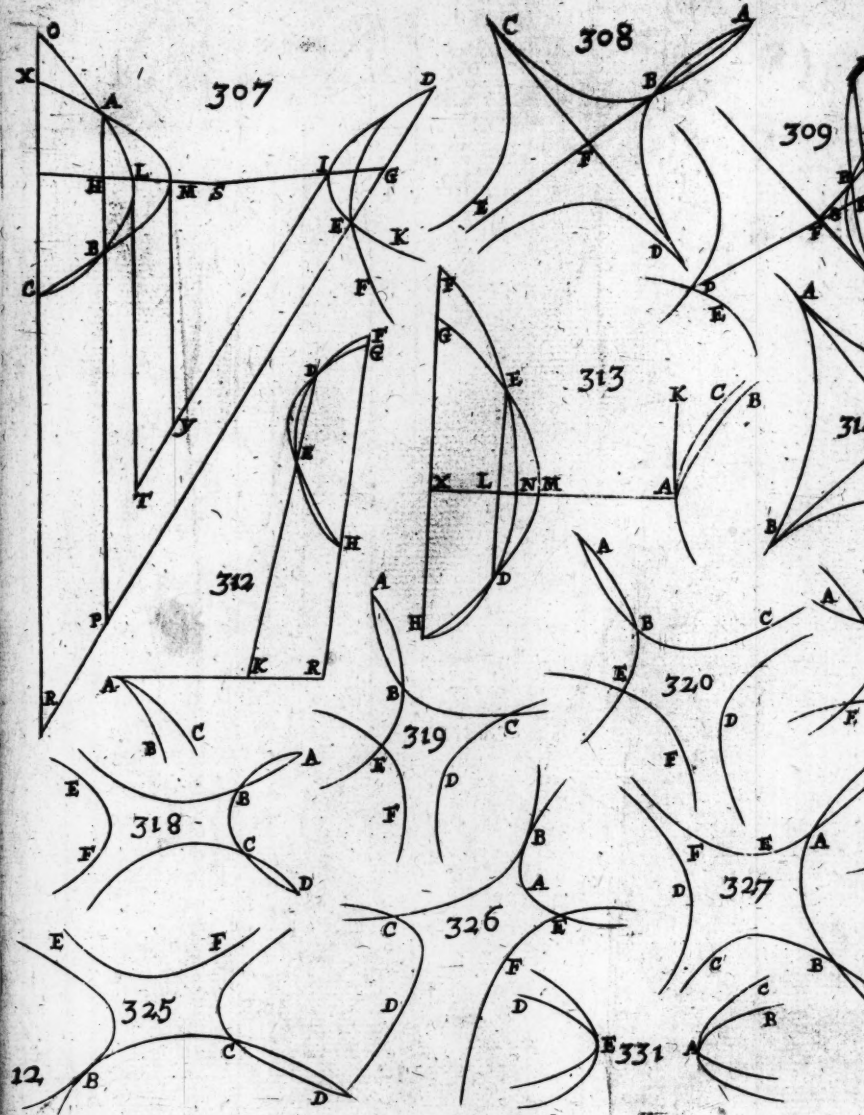
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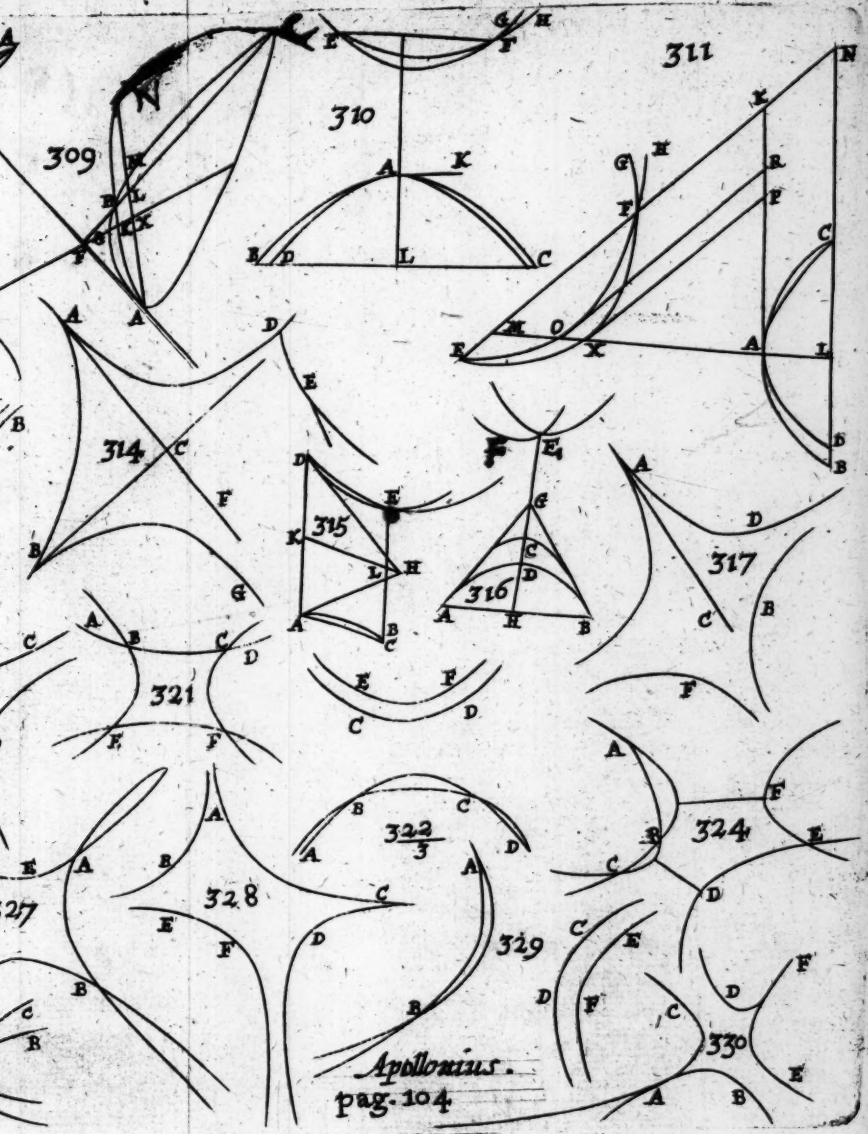

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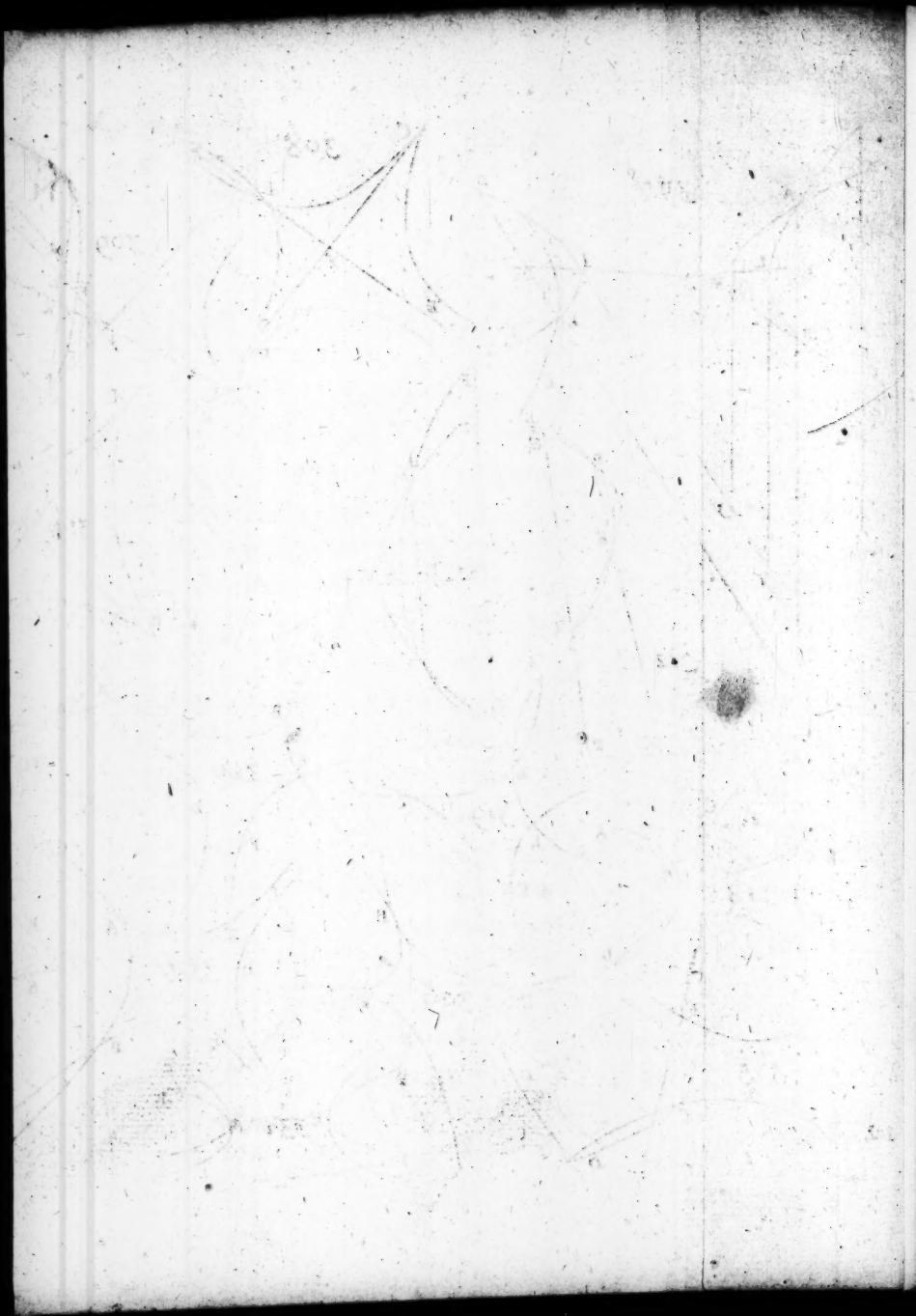

ERRATA sic corrigantur:

Pag.	Lin.	In Argumento.
2	8	A *pro* B.
2	13	X *pro* H & G.
3	2	Z vel Y *pro* X.
8	*pe.*	C F H *pro* F G M.
9	19	B M *pro* D M.
10	34	F N O L *pro* P X O L.
11	3	A G *pro* H G.
16	10	F E ⊏ D E *pro* G F ⊏ D E.
16	17	DEq. EB x BA :: Tq. Mq *pro* DEq. EB x AE :: Mq. Tq.
19	11	C F *pro* C E.
20	*ul.*	C D *pro* F B.
21	4	Q.AE—EB *pro* Q.EB—AE.
22	28	B A *pro* D A.
23	4,5 6,7	A G *pro* A E.
30	1	L X *pro* L K.
31	1	*In margine*, D E *pro* D C.
31	14	$\frac{4DEq}{BX}$ *pro* $\frac{4FEq}{BX}$
31	26	B H *pro* E H.
35	5	F G *pro* O G.
35	6	F H *pro* O H.
37	11	C B *pro* A B.
37	*pe.*	Y Z *pro* V Z.
38		Fig. 49. *pro* 94.
41	13	B H *pro* B E.
43	*pe.*	B A C *pro* B A D.
45	25	X V *pro* X Y.
52	22	A, B *pro* A, F.
54	13	A E *pro* B E.
58	25	M C *pro* Z C.
59	*ul.*	majus *p.* o minus.
62	16	G I *pro* C I.
64	22	L E *pro* L I.
64	26	O *pro* I.
64	29	D C *pro* G D.
66	6	B X K *pro* B X R, & B F *pro* B E.
66	16	A F *pro* A G.
68	17	D B q *pro* D K q.
68	33	H F x F E *pro* H F x F D.
68	34	
69	22	G I *pro* G L.
69	28	T O *pro* I O.
70	4	sectionum *pro* contingentium.
70	8	A E *pro* A F.
70	10	L S H *pro* L S F. & A L N *pro* A L M.
74	22	*Deest* + *inter* A Eq & C X x X A.
75	*pe*	*Deest* + *inter* E X q & C E x E D.
77	18	E C *pro* F C.
78	21	*Citatur* 4 & 22.6 *pro* 19.5.
79	10	A B *pro* A D.
79	26	K O B *pro* K O P.
86	5	F A *pro* D F.
91	*ul.*	O P *pro* O D.
96	*pe.*	F G *pro* F M.
99		*Ult. duæ lineæ ita corriganturr*, FN.FL::NK.KL & (ob sect. AMD) NK.KL :: NM . ML quar FN.FL::NM.ML *Q.e.d.*
102	8	A D, B G *pro* A C, B C.

In Citationibus.

Prop. 9. *nota* f, *pro* 17.6 *lege* 16.6. *pr.* 16. *n.* 2, 13 hujus *l* 12 hujus. *pr.* 20. *n.* c, 3.6 *l.* 1.6. *pr.* 33. *n.* c, 4.1 4.2. *pr.* 38 *n.* c, 21 hujus *l.* 37 hujus. *pr.* 44 *n.* 2, 31 hujus *l.* 37 hujus. *pr.* 45. *n.* f, 7.1 *l.* 7.5. *pr.* 45. *n.* g, 4.1 *l.* 4.6. *pr.* 51. *n.* m, 7.1 *l.* 7.5. Libri I.

Prop. 2. *nota* g, *pro* 6.5 & 3 ax. 1 *lege* 6 & 5.2. *pr.* 4. *n.* g, 53. hujus *l.* 53.1 hujus. *pr.* 48. *n.* g, *deest hæc citatio* (17 ax. 1.) Libri II.

Prop. 4. *nota* a, *pro* cor. 44 hujus *lege* cor. 15.2 hujus. *pr.* 8. *n.* k, 8.1 *l.* 4.3 hujus. *pr.* 16. *n.* d, & *pr.* 16, 17, 18, 19. *n.* b, *in singulis pro* 16.5 *l.* 4.6. *pr.* 20. *n.* c, 16.3 *l.* 4.6. *pr.* 21. *n.* b, 46.5 *l.* 4.6. *pr.* 22. *n.* k, 2.6 *l.* 6.2. *pr.* 23. *n.* b, 16.5 *l.* 4.6. *pr.* 23. *n.* d, *l.* facilè deducitur ex 15.3 hujus. *pr.* 33. *n.* d, 2.6 *l.* 6.2. *pr.* 34 *n.* e, 15.6 *l.* 16.6. *pr* 37. *n* b, 49 & 51. hujus *l.* 2 & 11.3 hujus. *pr.* 44. *n.* b, & *pr.* 45. *n.* c, *pro* 15.6 *l.* 16.6. *pr.* 56. *n.* p, ex 2 vel 4.8 *l.* 2 vel 4.3. Libri III.

Prop. 14 *nota* 2, *pro* cor. 14 hujus *lege* 5 hujus *pr.* 15. *n.* b, 36.1 hujus *l.* 39.3 hujus. *pr.* 21. *n.* a, 6 hujus *l.* 8 hujus. *pr.* 21. *n.* b, 15 hujus *l.* 17 hujus. Libri IV.

In Schematis.

Fig. 14. *deest linea* F G. fig. 30 *deest* D *ubi* E H *occurrit sectioni*. fig. 136. *pro* H *lege* B. fig. 148 *deest* D *ubi lineam* A C *bisecat* K B. fig. 176. *deest* E. fig. 194. *deest ubi* Y *ubi* L X *diametro occurrit*. fig. 276. *deest* E *ubi* D K *sectioni occurrit*.

			In Argumento.		
2	2		I pro B.	15	A E pro B E.
2	12		Y pro H & G.	15	M C pro Z C.
3			Z vel .		
8	pe		C H pro O M.		
9	15		B M pro D M.		
10	34		B N O L pro D X O L.		
11	3		A G pro H G.		
16	19		E L — D B pro O L — D E		
17			O B, E L × B A : T d . M e p r ,		
			D B d . E B × A B : : T d . M e p r ,		
19	11		C F pro C B.		
21	41		C G pro F B.		
21			G — E B pro Q E — A B		
22	28		B A pro D A.		
23	43		A G pro A E.		
25	2		L Y pro L K.		
31	1		D e m o n s t r a . D C pro D C.		
31			L D E d . L B d .		
31	14		B Z pro B X		
31	24		B H pro E H.		
35	5		F G pro O G.		
35	6		E H pro O H.		
35	11		C G pro A C.		
37	10		Y Z pro A Y.		
41	11		B H pro E B.		
43	pe		B A C pro B A D.		
45	21		Y V pro X Y.		
47	21		A , B pro A E.		
			A B , B C pro A C , B C .	103	1

In Demonstratione.

1	15		I pro B.	15	A E pro B E.
2	12		Y pro H & G.	15	M C pro Z C.
3			Z vel .		
8	pe		C H pro O M.		
9	15		B M pro D M.		
10	34		B N O L pro D X O L.		
11	3		A G pro H G.		
16	19		E L — D B pro O L — D E		
17			O B, E L × B A : : T d . M e p r ,		
			D B d . E B × A B : : T d . M e p r ,		
19	11		C F pro C B.		
21	41		C G pro F B.		
21			G — E B pro Q E — A B		
22	28		B A pro D A.		
23	43		A G pro A E.		
25	2		L Y pro L K.		
31	1		D e m o n s t r a . D C pro D C.		
31			L D E d . L B d .		
31	14		B Z pro B X		
31	24		B H pro E H.		
35	5		F G pro O G.		
35	6		E H pro O H.		
35	11		C G pro A C.		
37	10		Y Z pro A Y.		
41	11		B H pro E B.		
43	pe		B A C pro B A D.		
45	21		Y V pro X Y.		
47	21		A , B pro A E.		
			A B , B C pro A C , B C .	103	1